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Identification of the Vistula Mounting tower model using measured modal data

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ABSTRACT

This paper discusses the problem of parametric identification of a historic masonry tower model. The tower tends to lean and its foundation stiffness is a concern to authorities. The authors identified a few modal characteristics of the tower, natural frequencies and mode shapes. Based on the first mode shape identified it is known that the structure behaves like a stiff solid on elastic foundation. Thus, a simple, five parameter plane model is taken into consideration. The unknown parameters are identified to be the solution to an optimisation problem, which involves using the sensitivity analysis and scatters of the modal identification. A hierarchical process is formulated, where two natural frequencies are assumed to be the input data. In this approach, the number of unknown parameters increases incrementally, and the process changes from even-posed to under-posed successively. Such approach allows one to control the final under-posed identification problem and leads to an increasingly better solution.

1. Introduction

This paper discusses the problem of parameters identification of the Vistula Mounting tower model. A mathematical model can be a source of information about structural behaviour, in addition to observations and *in situ* measurements. However, the model must be well-posed. In mathematical modelling several problems must be considered and some decisions should be taken. First of all, the type of the model, its size and the governing equations must be specified in order to properly represent the structure. Secondly, some variables describing the structure's behaviour must be quantifiable and measured. They will serve as the data for model validation. Finally, a method of model parameters identification must be chosen.

Nowadays, when measuring tools are highly developed, it is convenient to use modal characteristics of a structure as reference data for modelling. Those characteristics can describe both the global and the local behaviour of a structure. Thus a model can be validated on different levels. Experimental modal identification belongs to the class of inverse problems. The modal characteristics are determined through measured structural response to known (i.e. generated signals especially) or unknown (i.e. ambient) excitations. There are a lot of methods of experimental modal identification, many of which are described in the book [1]. A review of different experimental techniques for obtaining modal parameters of concrete structures is presented in [2]. In the book [3] a background for the peak picking method is given. A practical example of the modal model identification of a 200 MW steam turbine foundation, working in a large power station is described in [4]. The model identification is based on measured response of the system to operating excitations. Another case is described in the paper [5], where application of different modal identification techniques for collecting ambient response data from dynamic tests of cable-stayed bridge is discussed.

Parametric identification of a model is also an inverse problem. If response of a model, i.e. the modal characteristics, displacements, time histories, etc., can be compared to the response measured *in situ*, then the model parameters can be identified. For instance, the paper [6] describes an example, where concrete gravity dams were identified to predict dynamic characteristics of an existing concrete gravity dam with an empty reservoir. An experimental examination of a hybrid method for identifying both structural parameters and ground motion of earthquake-excited multi-storey buildings is described in [7]. Another case, concerning bridge finite element model updating, is presented in [8].

In this article a problem of modelling and parametric identification of a historic masonry tower in the Vistula Mounting Fortress in Gdansk (Fig. 1) is described. The tower, in its original form, dates back to the 15th century. It was erected as a lighthouse and a defensive building. It was damaged several times in military conflicts. Nowadays it is 22.65 m high, and its external diameter is 7.7 m. The structure has seven floors. Concrete ceilings are reinforced. Its walls were built using masonry and were restored at different times. The average wall thickness is 1.25 m. The tower was founded on weak and layered subsoil. The foundations were made of boulders and lie just below the ground level. Conservation works in

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Fig. 1. Vistula Mounting Fortress with the tower in the centre.

the Vistula Fortress have continued for the last 20 years. In May 2011 the Fortress was opened for visitors.

There are still, however, dilemmas concerning the tower. One of them concerns stiffness of the tower foundation because the building tends to lean. The author's task is to propose a relatively simple model of the tower and estimate foundation stiffness.

The overall behaviour of the tower can suggest a type of a suitable mathematical model. In order to determine it, dynamic measurements were taken and some modal characteristics have been identified. Signals measured at several structural points during ambient excitation have been utilized. It should be emphasised that only weak ambient excitations caused by the river and wind are possible to use in the modal identification process because of bad tower condition.

The type of a tower model was selected based on the form of the first mode shape. Since a considerable rotation – in comparison to the tower structural deformation – about the tower base is observable, a rigid solid body resting upon elastic foundation is considered to be a good approximation of the structure. Natural frequencies of the first and the second coplanar mode shapes, and two coordinates of the first mode shape are used as the data in the model parametric identification. In order to solve the problem of model identification, a least square error function was formulated as the objective function. To minimize it, an iterative procedure with implication of the sensitivity analysis was employed.

In this engineering case the authors wish to propose a hierarchical approach to solve the under-posed problem. Important elements of this task are scatters of the structure's measured modal characteristics. They are used to accurately define the optimisation problem.

2. Identification problem

2.1. Experimental modal identification

The difficulty of dynamic identification of masonry buildings is caused by low-energy vibrations of such structures (see [9]). The peak picking method (see [3]) was used for modal identification of the tower. The method is suitable for any signal, also for low-energy ones. It was used with success for other masonry tower examinations, for example for the historic masonry bell-tower, adjacent to the Cathedral of Monza in Italy [10]. The biggest advantage of the method, however, is the possibility of determining statistical errors of identified modal characteristics. This feature of the method was useful for this investigation.

The mode shapes errors arise from the fact that only estimates of the auto-spectra, which are basic functions in the peak picking method, can be calculated. Real values of the functions could be obtained for signals infinite in time and that is practically impossible. The estimates are affected by statistical errors, bias ε_b and random ε_r, which give a final error $\varepsilon = \varepsilon_b + \varepsilon_r$. They are presented in [3] and [11]. The formulae are:

$$\varepsilon_b[\widehat{G}_{pp}(f)] \approx \frac{\Delta f^2}{24}\left[\frac{(G_{pp}(f))''}{G_{pp}(f)}\right], \tag{1}$$

$$\varepsilon_r[\widehat{G}_{pp}(f)] \approx \frac{1}{\sqrt{n_d}}, \tag{2}$$

where $\widehat{G}_{pp}(f)$ is the estimate of auto-spectrum calculated for signal measured in a structural point p, Δf denotes the frequency resolution of the analyzed spectra, $(G_{pp}(f))''$ is the second derivative of the $G_{pp}(f)$ function and n_d is a number of $p(t)$ signals analyzed.

Coordinates of a mode shape associated with the resonant frequency f_m are calculated according to the formula (3) (see [3]):

$$\phi_p(f_m) = \sqrt{\frac{\widehat{G}_{pp}(f_m)}{\widehat{G}_{rr}(f_m)}}, \tag{3}$$

where $\phi_p(f_m)$ denotes the estimated mode shape coordinate at discretization point p and $\widehat{G}_{rr}(f_m)$ is the auto-spectrum value for f_m, calculated for a signal $r(t)$, measured at the structural reference point r. Thus, according to the rules of error transfer the following statistical error of the mode shape coordinates is identified:

$$\varepsilon[\phi_p] = \frac{1}{2}\left(\varepsilon[\widehat{G}_{pp}] + \varepsilon[\widehat{G}_{rr}]\right) \tag{4}$$

The error of the measured natural frequencies has two components: the digitalisation error equal to the half of the spectrum resolution, and the random error calculated using dispersion of the measured resonant frequencies.

Acceleration of points selected across the tower were measured during ambient vibrations. Wind and water waves from the nearby situated river (Fig. 1) caused major environmental excitation. The measuring points were arranged along two opposite walls at the tower height on nine levels (Fig. 2). Accelerations in two horizontal directions, East–West (parallel to the wall surfaces) and North–South (perpendicular to the wall surfaces) were recorded at each point. Thus, 36 measuring points were set. A twelve-channel PULSE 3650C measuring system was used. Four series of measurements were carried out, in which four sensors were constantly in the same positions, in the reference points, situated at the level 11.05 m above the ground level. The signals collected in the reference points were used for scaling measurements made in different series to the same order. Each measurement took 1024 s, 256 samples were collected per second, so each signal consisted of 262144 samples.

According to rules of the peak picking modal identification method, the auto- and cross-spectra of the time series, the coherence functions for various pairs of signals and the phase shifts between them were calculated. On that background only one resonant frequency of the tower was identified using signals measured across the North–South plane, whereas three were determined using time series measured in the East–West direction. An example cross-spectrum of signals measured at the tower top at two structural points situated on opposite walls is presented in Fig. 3. Nature of related mode shapes was also specified using the analysis of phase shifts between signals measured at different