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Deriving stress–strain relationships for steel fibre concrete in tension from tests of beams with ordinary reinforcement

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ABSTRACT

One of the most critical points in the theory of steel fibre reinforced concrete (SFRC) is quantifying the residual stresses in tension. Due to concrete interaction with fibres, a cracked section is able to carry a significant portion of tensile stresses, called the residual stresses. Because of a great diversity in the shape and aspect ratio of fibres and, consequently, varying bond characteristics, there are no currently available reliable constitutive models. In present practices, residual stresses needed for strength, deflection and crack width analysis are quantified by means of standard bending tests. However, such tests require relatively sophisticated and expensive equipment based on the displacement-controlled loading. Besides, the test results are highly scattered. This paper investigates an alternative approach for defining the residual stresses. The approach aims at deriving equivalent stress–strain relations of cracked tensile concrete using test moment–curvature relationships of flexural concrete members with ordinary reinforcement and steel fibres. Tests on eight lightly reinforced beams (reinforcement ratio 0.3%) with different contents of steel fibres (0%, 0.5%, 1.0%, and 1.5% by volume) have been carried out. Based on the proposed technique, equivalent stress–strain relations were defined for each of the beams and further used for curvature and crack width analyses.

1. Introduction

Two main disadvantages of concrete as a structural material are its low tensile strength and brittleness. The inclusion of steel fibres may significantly improve mechanical properties of concrete such as ductility and residual load carrying capacity (toughness) [1]. The issue of quantifying the residual tensile strength (stresses) for a cracked section is one of the most critical points in the theory of steel fibre reinforced concrete (SFRC) [2–8]. To model the behaviour of SFRC in tension, the contribution of fibres can be introduced in two different ways: (1) by addition of a term (dependent on the fibre geometry and orientation) to represent the fibre stress and fibre-reinforcement ratio in a crack and (2) by considering SFRC as a homogeneous material with higher toughness characterized by the ability of a cracked section to resist a substantial fraction of tensile strength. The latter approach, considered in the present study, due to its simplicity and numerical effectiveness is much more widely used than the first one.

In present practices, the residual stress is quantified by means of bending tests on notched or un-notched specimens (600–700 mm in length). Based on these tests and standard techniques (*RILEM* [9], *DBV* [10], etc.), the first crack strength and the residual stresses are defined. These parameters are further used for the strength, deflection and crack width analysis of SFRC members. The standard techniques are always accompanied by a large scatter of the test results [8,11–15] reaching over 30%. The scatter is closely related to variation in the number of fibres across the fracture plane [16].

Due to a great diversity in the shape and aspect ratio of steel fibres and, consequently, varying bond characteristics, there are no reliable constitutive models of residual stresses available until present time [17]. This, as a consequence, limits application of steel fibre reinforced concrete. In numerical simulation, the post-cracking behaviour is modelled either by a stress–strain relationship [2,4,18–20] or a stress–crack width law [6,21–23]. In less sophisticated applications, a constant value of the residual stress can be assumed [3,9].

Average stress–average strain relationships in tension can be obtained using an innovative inverse technique proposed by Kaklauskas and Ghaboussi [24]. The innovative technique is based on the layer section model [25] and the smeared cracking conception. For a given experimental moment–curvature diagram, the equivalent stress–strain relationship is progressively computed for the extreme tension fibre of the concrete section. Recently, the inverse technique was modified [26] to eliminate the shrinkage effect from the test data of flexural reinforced concrete elements [27,28].

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The present study extends application of the inverse technique to analysis of SFRC elements. It aims at deriving the equivalent stress–strain relations for steel fibre reinforced concrete in flexural tension using test data of concrete beams reinforced with steel bars and fibres. The paper reports results of the experimental and numerical investigation of deformation and cracking behaviour of SFRC flexural members. The experimental part includes tests on eight lightly reinforced beams (longitudinal reinforcement ratio 0.3%) with different contents of steel fibres (0%, 0.5%, 1.0%, and 1.5% by volume). Based on the proposed inverse technique, the equivalent stress–strain relations for concrete in tension were defined for each of the beams and further used for the curvature and the crack width analyses.

2. Constitutive analysis of the flexural members

Present investigation is aimed at developing a numerical procedure for the residual strength analysis of SFRC in tension using moment–curvature relationships of beams with ordinary reinforcement and steel fibres. The *inverse* procedure is based on the *direct* technique, the *Layer* section model [25] and uses the following approaches and assumptions.

2.1. Approaches and assumptions

(1) Fibre concrete is considered to be a homogeneous material.
(2) The smeared crack approach is adopted. As yielding of reinforcement occurs in a single section, this assumption limits the validity of the inverse analysis until the load corresponding to the yielding point.
(3) Linear strain distribution within the depth of the section is assumed.
(4) All concrete layers in the tension zone follow a uniform stress–strain law. This assumption allows reducing the dimension of the inverse problem solution to a single nonlinear equation.

2.2. Direct technique: Moment–curvature analysis

Let us consider a doubly reinforced concrete member subjected to an external bending moment M_{ext}. A cross-section for such member is presented in Fig. 1a. As shown in Fig. 1b, the cross-section is divided into n longitudinal layers. They correspond to either concrete or reinforcement. Thickness of the reinforcement layer is taken from the condition of the equivalent area. The *direct* analysis needs to assume material laws for the reinforcement (Fig. 1f) as well as for the concrete in compression (Fig. 1g) and in tension (Fig. 1h). Following the assumption (2), the latter diagram is limited to the strain in the extreme tensile concrete layer ε^* corresponding to the start of yielding of reinforcement.

Curvature κ and strain ε_i at any layer i (see Fig. 1d) can be calculated by the formulae:

$$\begin{aligned} &\kappa = \tfrac{M_{ext}}{IE}; \quad \varepsilon_i = \kappa(y_i - y_c); \quad y_c = \tfrac{SE}{AE}; \quad AE = b\sum_{i=1}^{n} t_i E_{i,\sec}; \\ &SE = b\sum_{i=1}^{n} t_i y_i E_{i,\sec}; \quad IE = b\sum_{i=1}^{n}\left[t_i^3/12 + t_i(y_i - y_c)^2\right]E_{i,\sec}. \end{aligned} \tag{1}$$

Here AE, SE and IE are the area, the first and the second moments of inertia multiplied by the *secant* modulus $E_{i,sec}$. Other notations are evident from Fig. 1b.

For the given strain ε_i and the constitutive law (see Fig. 1f, g and h), stress σ_i is obtained. The secant deformation modulus $E_{i,sec} = \sigma_i/\varepsilon_i$ is determined ($i = 1 \cdots n$). The analysis is performed iteratively until convergence of secant modulus at each layer is reached. Fig. 1d and e illustrate strain and stress distributions within the *Layer* section model. The calculation is terminated when the ultimate load step is reached.

It should be noted that the assumed number of layers n might have influence on the calculation results. The recommended number $n = 200$ [28] most effectively secures the computational efficiency in terms of convergence and accuracy.

2.3. Inverse technique: Deriving equivalent stress–strain relationships of SFRC in tension

Unlike the *direct* analysis, aiming at prediction of structural response using a specified constitutive model, the *inverse* analysis has an objective to determine parameters of the model based on the response of the structure. Present investigation deals with moment–curvature measurements of beams with ordinary reinforcement and steel fibres and aims at selecting the equivalent stress–strain model of SFRC in tension. The selected model will allow predicting the same moment–curvature response as it was obtained at the tests.

Kaklauskas and Ghaboussi [24] have formulated the principles of the *inverse* technique for deriving average stress–average strain relationships of tensile concrete using the test data of reinforced concrete flexural members. In the present study, the inverse technique modified by Gribniak et al. [26] has been employed for deriving the constitutive laws of SFRC in tension. The analysis utilizes the *Layer* section model (see Fig. 1b) and is based on the concept of a progressive calculation of the stress–strain relationship for the extreme tension layer of the section (see Section 2.1). For a given load increment, an initial value of the secant deformation modulus of SFRC in the extreme tensile layer is assumed and a curvature is calculated using the *direct* technique (see Section 2.2). If the calculated and experimental curvatures differ, methods of iterative analysis are used to correct the secant modulus.

Fig. 2 presents a flow chart of the *inverse* technique. Based on geometrical parameters of the cross-section, the *Layer* section is composed. Stress–strain material laws for steel and compressive concrete are assumed. Computations are performed iteratively for an incrementally increasing bending moment. At each moment increment i, an initial value of the secant deformation modulus of stress–strain relationship under derivation is assumed equal to zero ($E_{i,0} = 0$). Curvature $\kappa_{th,i}$ is calculated by the *direct* analysis. If the agreement between the calculated and the experimental curvature $\kappa_{obs,i}$ is not within the assumed tolerance Δ, i.e. Condition **1** is not fulfilled (see Fig. 2), the analysis is repeated using the hybrid Newton-Raphson and bisection procedure [27] until Condition **2** is satisfied. At each iteration k, a secant deformation modulus $E_{i,k}$ is calculated. If the solution is found, i.e. Condition **1** is satisfied, the obtained value of $E_{i,k}$ is fixed and used for next load increments. If the limit iteration number is exceeded ($k > N = 30$), the calculated $E_{i,30}$ is rejected, meaning that the secant deformation modulus E_i is not defined at the moment increment i, and the analysis moves to the next load step. The calculation is terminated when the ultimate loading step is reached (Condition **3**). The analysis results in the derived equivalent stress–strain relationship of SFRC in tension.

As noted, the *inverse* analysis is performed incrementally, using the constitutive law obtained at previous loading stages. Consequently, the error made at a given moment increment will have influence on the shape of the remaining part of the constitutive law under the construction. Although in absolute terms being similar at all loading stages, the errors of curvature measurements at early stages have much higher relative effect on the resulting stress–strain curve. Therefore, particular care should be taken in the early stages of the analysis associated with small curvatures. As prior to cracking concrete both in compression and tension