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A numerical model for wind-borne compact debris trajectory estimation: Part 1 – Probabilistic analysis of trajectory in the proximity of tall buildings

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ABSTRACT

This paper describes the development of a numerical algorithm for investigating the probability of wind-borne compact debris, impacting on the vertical façade of tall buildings. A uniform wind field with constant velocity was employed; turbulence effects were neglected.

The trajectory of compact debris was estimated by solving the three-dimensional (3D) equations of motion of the object, immersed in uniform wind field; simulation results were compared to experimental data, derived from literature. Random parameters were selected to estimate the trajectory and to derive "Iso-probability Impact Contours", using Monte Carlo simulation. These contours describe the probability associated with "randomly flying" debris, as they impact against a cladding on the vertical façade, conditional on the initial position of the object relative to the structure. These contours were developed for two different cases, "Quasi-2D" and "Fully 3D", to account for wind directionality.

Moreover, a "Universal Probability Curve", describing the probability of impact for compact objects against the façade of a benchmark tall building, was developed in two-dimensional (2D). It was proven that this curve is universal and can be used independently of wind velocity.

1. Introduction

Wind-resistant design of tall buildings, based on performance of the structural and non-structural elements, has recently received relevant attention by researchers (e.g., [1]). Damage, induced by wind-borne debris, can play a significant role if the objective of the structural building design includes the assessment of "vulnerability" of vertical façades against wind hazards. "Wind-borne debris" (e.g., [2]) are flying objects, transported by wind during extreme events such as hurricanes. Debris can cause serious damages to the cladding elements on the façade of tall buildings, especially in urban areas.

Traditionally, the study of wind-borne debris trajectory has been a concern for special structures such as nuclear power plants and for tornadic winds [3,4]. For residential buildings, modeling of wind-borne debris trajectories has attracted the attention of the research community in more recent years and especially in the United States, after damage attributed to debris was observed during the landfall of Hurricanes Katrina and Ike in urban areas [5,6]. Most numerical and experimental work investigated the prediction of the debris trajectory, predominantly in two dimensions (2D) (e.g., [7,8]); more recently attention was devoted to three dimensions (3D) (e.g., [9,10]).

Also, statistical models for the prediction of wind-borne debris damage risk have been developed. However, the objective of most techniques, currently available, is the evaluation of the damage risk at the "regional scale" (i.e., an overall estimation for a residential area) and are often applied to vulnerable low-rise buildings (e.g., [11]). Less attention has been paid to the "local scale" problem, for example the need for analyzing the risk for the cladding of a large building, especially a tall structure. Tall buildings are particularly exposed to debris damage because of the large glass openings on their façade.

Four important components need to be considered for the modeling of the trajectory and the prediction of the damage produced by wind-borne debris on building structures. These components are: wind field [12,13], debris generation [14,15], debris trajectory [2,7,8,16–18] and failure mechanism of non-structural elements (e.g., glass windows) due to impact [11,19].

A model for the debris flight in uniform wind field was utilized in this study to simulate "compact debris" trajectory. A compact debris is defined [20] as an object of small dimensions, negligible moment of inertia compared to its mass, and for which the aerodynamic effect of lift force can usually be neglected. The equations of motion were numerically solved by using Runge–Kutta method in the time domain in 2D and 3D. Trajectories of both "cubes" and "spheres" were assessed, as these two types well represent the category of compact debris. An impact risk model was also developed to analyze "damage" risk for the glass façade of a 183 m tall

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building. Monte Carlo simulation [21] was used under the assumption that the fundamental physical parameters, controlling the flight trajectory, are random variables. The debris impact probability against selected areas of the building envelope was determined.

The main contribution of this investigation is the generation of "Iso-probability Impact Contours", based on the random trajectories, determined by numerical simulation. These contours describe the probability associated with "randomly flying" debris, as it impacts against the façade, conditional on the mean wind speed magnitude and the relative initial object position in the proximity of the structure.

Moreover, a "Universal Probability Curve", describing the probability of impact for compact objects against the façade of a benchmark tall building, was developed in two-dimensional (2D). It was proven that this curve is universal and can be used independently of wind velocity.

2. Theoretical background: dynamic trajectory equations for compact debris flight

The wind field was assumed as uniform with constant mean velocity U, independent of the elevation. The horizontal mean-wind direction angle (yaw) was denoted by Φ. Eq. (1)–(3) below describe the motion of compact debris in 3D space (X, Y, Z) and in dimensionless form. If the position of the debris in 3D space is described by dimensional coordinates x, y, z, as a function of time t, the dimensionless dynamic equations are (e.g., [8,9]):

$$\frac{d^2\bar{x}}{d\bar{t}^2} = KC_D\sqrt{\left[(\cos\Phi - \bar{u}_x)^2 + (\sin\Phi - \bar{u}_y)^2 + \bar{u}_z^2\right]}(\cos\Phi - \bar{u}_x), \qquad (1)$$

$$\frac{d^2\bar{y}}{d\bar{t}^2} = KC_D\sqrt{\left[(\cos\Phi - \bar{u}_x)^2 + (\sin\Phi - \bar{u}_y)^2 + \bar{u}_z^2\right]}(\sin\Phi - \bar{u}_y), \qquad (2)$$

$$\frac{d^2\bar{z}}{d\bar{t}^2} = KC_D\sqrt{\left[(\cos\Phi - \bar{u}_x)^2 + (\sin\Phi - \bar{u}_y)^2 + \bar{u}_z^2\right]}(-\bar{u}_z) - 1. \qquad (3)$$

In Eqs. (1)–(3) $K = U^2\rho_{air}A/(2\text{ mg})$ is the Tachikawa number [7,17], i.e., the ratio between aerodynamic forces and gravity; the dimensionless position variables are $\bar{x} = xgU^{-2}$, $\bar{y} = ygU^{-2}$, $\bar{z} = zgU^{-2}$ with time $\bar{t} = tg/U$; $\bar{u}_r = u_r/U$ (with $r = x, y, z$) are velocity components; C_D is the drag coefficient assumed as a constant parameter, independent of relative angle of attack between moving object and velocity, as a first approximation for a compact object; A is the reference (projected) area of the object, while m is the debris mass with g being the gravity acceleration and $\rho_{air} = 1.25\text{ kg/m}^3$. No variation of mean-wind velocity (U) with coordinate z was assumed in this study.

3. Deterministic trajectory model: validation (2D)

In 2007, an empirical equation was proposed to estimate debris trajectory [8]; two relationships, also based on a series of observations in wind tunnel, were derived for cubes $K\bar{x} = 0.4(K\bar{t})^2 - 0.04(K\bar{t})^3 - 0.05(K\bar{t})^4 + 0.01(K\bar{t})^5$, and for spheres $K\bar{x} = 0.25(K\bar{t})^2 + 0.08(K\bar{t})^3 - 0.1(K\bar{t})^4 + 0.006(K\bar{t})^5$, appropriate during early flight stages. These empirical equations will be referred to as "Lin's equations" in the remainder of this section. A series of wind tunnel tests [8] recommended appropriate values for C_D of both cubes and spheres.

In this section, the equations of motion were numerically integrated by fifth-order Runge–Kutta algorithm [22] and compared to Lin's equations to identify a suitable drag coefficient. Moreover, the time step and tolerance of the numerical algorithm were calibrated to successfully reproduce the empirical relationships.

In order to compare these results with the empirical equations (above) and for calibrating the parameters of the Runge–Kutta algorithm, the 3D system of equations must be simplified to 2D: $\Phi = 0$ was used in Eqs. (1)–(3) to obtain a uniform wind field with velocity U parallel to the x direction; zero initial position and velocity for components other than x were set to zero. Preliminary simulations at $U = 30$ m/s suggested that a time step equal to 0.001 s and tolerance equal to 0.001 m were acceptable for accuracy and to minimize computational time.

An example of numerical trajectory in dimensionless form is shown in Fig. 1 for cubes and spheres with Tachikawa number $K = 7$. As shown in Fig. 1 there is a good agreement between the simulated trajectory and the empirical equation based on $K = 7$ and a constant C_D. Similar results (not shown for brevity) were observed for other combinations of K and C_D. These results validated the numerical algorithm for deterministic analyses and enabled the further implementation of the probability-based studies.

4. Probability-based modeling of flight trajectory

A damage risk model was developed, which makes use of "Iso-probability Impact Contours". These lines may possibly provide an estimation framework for assisting a rational decision by designers about the need for protecting the glass façade of a tall building against potential wind-borne debris damage. Predictions were carried out at the "local scale" of the cladding elements, and employed a specific high-rise structure as a pilot example.

For an accurate generation of the Iso-probability Impact Contours, the main physical quantities, utilized in Eqs. (1)–(3) and combined with the initial flight conditions, were assumed as

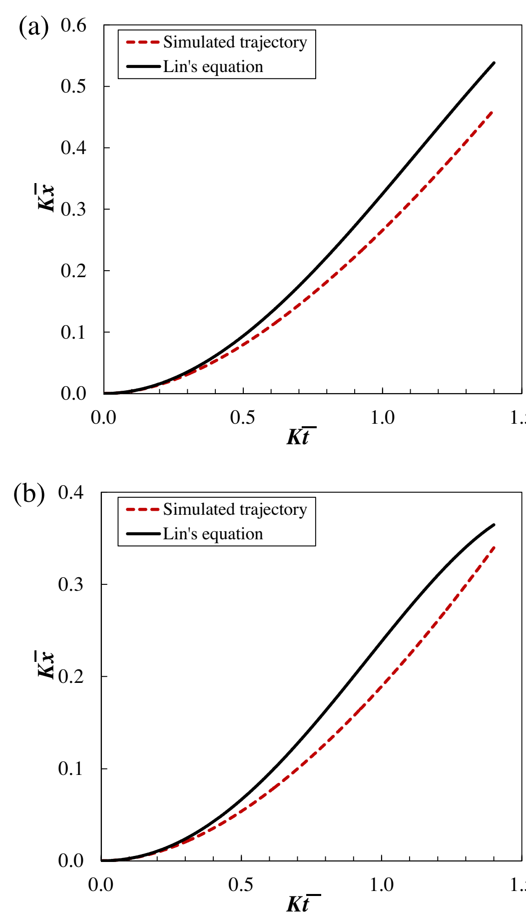

Fig. 1. Comparison between simulated horizontal trajectory and empirical equation [8] for an object with $K = 7$: (a) cubes with $C_D = 0.8$; (b) spheres with $C_D = 0.5$.