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Analyses of plastic hinge regions in reinforced concrete beams under monotonic loading

Xue-Mei Zhao, Yu-Fei Wu *, A.Y.T. Leung

Department of Civil and Architectural Engineering, City University of Hong Kong, Hong Kong

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ABSTRACT

Due to the high complexity and difficulty involved, the behavior of plastic hinge of reinforced concrete members has been previously investigated experimentally. This work investigates the plastic hinge analytically, using the finite element method. Lengths in the plastic hinge region involving rebar yielding zone, concrete crushing zone and curvature localization zone are studied systematically, under different parameters. The results show that none of the existing empirical models is adequate for prediction of plastic hinge length. As high non-linearity occurs in plastic hinge zones, the phenomenon of chaos is observed, for the first time, in flexural response of reinforced concrete structures.

1. Introduction

For decades the plastic hinge of reinforced concrete (RC) members where plastic deformation is concentrated has been an interesting and complicated subject for researchers and engineers. The performance of a plastic hinge is critical to the load carrying and deformation capacities of flexural members. In the meantime, plastic hinge region of RC flexural members is a critical zone needing intensive care to prevent failure of structural members from extreme events such as earthquakes. Therefore, quantification of the plastic hinge zone is important not only for design of new structures but also for rehabilitation of old structures. For example, construction of retrofitting work by external FRP jacketing needs to know the extent of concrete crushing zone and significant rebar slipping zone (significant slip causes concrete cover spalling) so that a sufficiently long jacket is constructed.

Although numerous empirical equations have been proposed in extant literature for prediction of plastic hinge length L_p, accuracy (or even the definition) of L_p remains an open issue, yet to be addressed adequately. A combination of three phenomena, namely, the high concentration of compression strain around the section of maximum moment that complicates the notion of base curvature, the tension shift that invalidates the assumption of the plane section remaining plane, and the strain penetration that results in a fixed end rotation at the support, explains the difficulties of the problem [1].

Several well-known empirical models [2–13] have been proposed for L_p (Table 1). The conventional plastic hinge length L_p in Table 1 is considered as a virtual length over which a given plastic curvature is assumed to be constant for integration of cross-sectional curvatures along the RC member length, to solve the member's flexural deflection and plastic rotation capacity [14]. In this case, it may not be the physical length of the real plastic hinge region over which the actual plasticity spreads. Nevertheless, physical length of the real plastic hinge region is logically believed to have a certain intimate relationship with L_p.

High nonlinearity of materials and the interactions and relative movements between constituent materials in the plastic hinge zone greatly complicate the problem. As a result, studies of plastic hinges in RC members have so far been limited to experimental testing. However, the traditional way to investigate the problem through experimental testing is restricted by the time and cost involved in large tests. As finite element (FE) analyses become increasingly matured and with rapid increase in computer speed and storage capacity, it is becoming possible to simulate the highly complicated problems with the FE method (FEM). This work tries to investigate the plastic hinge region of RC beams in details through FE numerical simulations.

2. Finite element modeling and implementation

Commercial software DIANA is employed for FE simulations in this work. To obtain mesh objectivity, crack band model is adopted in the study, as it has been widely and successfully applied to RC structures for simulation of not only global response but also local

* Corresponding author. Tel.: +852 34424259; fax: +852 34427612.
E-mail address: yfwu00@cityu.edu.hk (Y.-F. Wu).

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Nomenclature

A	total area of the element in FE model
A_g	gross area of concrete section
A_s	area of tension reinforcement
b	width of beam
c_0	coefficient related to steel type, axial loading and concrete strength
c_1	0.7 for mild steel0.9 for cold-worked steel
c_2	$1 + 0.5(p/p_u)$
c_3	$0.9 - (0.3/23.5)(f_c - 11.7)$ (f_c in MPa)
d	effective depth of beam or column
d_b	diameter of longitudinal reinforcement
$d_{\max}$	maximum size of aggregates
E_c	Young's modulus of concrete
E_s	Young's modulus of steel
E_{sh}	hardening modulus of steel
f_c	concrete compressive strength
f_{co}	unconfined concrete compressive strength
f_s	steel stress
f_t	concrete tensile strength
f_y	yield strength of reinforcement
F	applied load
F_{cr}	load at crack of concrete
F_{yr}	load at yield
G_f^c	concrete fracture energy in compression
G_f^I	concrete fracture energy in tension
h	overall depth of cross-section
h_b	crack bandwidth
k	curvature
k_{yy}	average curvature within L_{sy} at first yield
L_{cs}	length of compression zone where $\varepsilon_c > 0.002$
L_{cc}	length of compression zone where $\varepsilon_c > 0.006$
L_g	distance over which average curvature or rotation is calculated
L_p	equivalent length of plastic hinge
L_{pc}	maximum length of curvature increment zone
L_{pcs}	length of significant curvature localization zone
L_{sh}	maximum length of rebar strain fluctuation zone
L_{sy}	maximum length of rebar yielding zone
M_y	bending moment at yield
M_u	bending moment at ultimate
p	ultimate axial load on the member allowing for bending moment when present
p/p_o	axial load ratio
p_o	$0.85(A_g - A_s) + f_yA_s$
p_u	ultimate load capacity of the member under axial force only
s	bond slip
z	distance from critical section to point of contraflexure
β_e	hardening or softening slope of load-displacement curve
ε_c	concrete compressive strain
ε_{co}	unconfined concrete compressive strain at peak stress
ε_s	steel strain
ε_{sp}	concrete spalling strain
ε_t	concrete tensile strain
ε_{20}	strain at which the stress reaches 20% of f_c after peak stress
σ_c	concrete compressive strength
σ_t	concrete tensile strength
θ	rotation of section
Δ	mid-span deflection
Δ_{cr}	mid-span deflection at crack of concrete
Δ_{yr}	mid-span deflection at yield
ρ_s	tensile reinforcement ratio
ρ_{sc}	compressive reinforcement ratio
τ	bond stress
τ_f	residual bond stress
$\tau_{\max}$	maximum bond stress

cracking patterns [15–19]. For simplicity, only simply supported beams under three-point bending and monotonic loading are studied with the two-dimensional model for half beam (Fig. 1). As flexural deformation dominates in plastic hinges, shear dominant members with small aspect ratios (ratio of shear span length to depth of beam) are not considered in this work.

2.1. Modeling of concrete

The three-node triangular isoparametric plane stress element is adopted for concrete elements in the study. The mesh size is generally to be 10–20 mm, which is close to the size of the aggregate.

Table 1
Empirical models for plastic hinge length.

Reference	Plastic hinge length (L_p)
Baker [2]	$c_0(z/d)^{1/4}d$ (for RC beams and columns)
I.C.E. Research Committee [3]	$c_1c_2c_3(z/d)^{1/4}d$
Sawyer [4]	$0.25d + 0.075z$
Corley [5]	$0.5d + 0.2\sqrt{d}(z/d)$ (for RC beams)
Mattock [6]	$0.5d + 0.05z$ (for RC beams)
Priestley and Park [7]	$0.08z + 6d_b$ (for RC columns)
Paulay and Priestley [8]	$0.08z + 0.022d_bf_y$ (for RC beams and columns)
Sheikh and Khoury [9]	$1.0h$ (for columns under high axial loads)
Coleman and Spacone [10]	$G_f^c/[0.6f_c(\varepsilon_20 - \varepsilon_c + 0.8f_c'/E_c)]$
Panagiotakos and Fardis [11]	$0.18z + 0.021d_bf_y$ (for RC beams and columns)
Bae and Bayrak [12,13]	$h\{[0.3(p/p_o) + 3(A_s/A_g) - 1](z/h) + 0.25\} \geqslant 0.25h$ (for columns)

2.1.1. Tensile behavior of concrete

The rotating smeared crack model with total strain concept is adopted for the nonlinear tension softening behavior of concrete [19,20]. Softening is assumed to be exponential, as illustrated in Fig. 2(a); details are available in DIANA User's Manual [21]. The main parameters for this softening curve are tensile strength of concrete f_t, tensile fracture energy G_f^I, and crack bandwidth h_b.

In this work, fracture energy G_f^I of concrete is calculated according to the CEB-FIP Model Code 1990 [22]:

$$G_f^I = G_f^0(f_c/10)^{0.7}, \tag{1}$$

where the value of G_f^0 depends on the maximum aggregate size $d_{\max}$; and the crack bandwidth is taken as $\sqrt{2A}$, in which A is the total area of the element. The maximum aggregate size is assumed to be 16 mm.

2.1.2. Compressive behavior of concrete

Mander et al.'s stress–strain model [23] is adopted for both confined and unconfined concrete. A typical stress–strain envelope for unconfined concrete is depicted in Fig. 2(b) and the corresponding equation is:

$$\sigma_c = \frac{f_{co}xr}{r - 1 + x^r}, \tag{2}$$