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Numerical pathologies in snap-through simulations

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ABSTRACT

Aircraft structures operating in severe environments may experience snap-through, causing the curvature on part or all of the structure to invert inducing fatigue damage. This paper examines the performance of beam and continuum nonlinear finite element formulations in conjunction with several popular implicit time stepping algorithms to assess the accuracy and stability of numerical simulations of snap-through events. Limitations of the structural elements are identified and we provide examples of interaction between spatial and temporal discretizations that affect the robustness of the overall scheme and impose strict limits on the size of the time step. These limitations need to be addressed in future works in order to develop accurate, robust and efficient simulation methods for response prediction of structures encountering extreme environments.

1. Introduction

Curved beams or panels can often be found as components of complex structures in civil, mechanical, and aerospace applications. They may experience snap-through causing the curvature on part or all of the structure to invert due to a large inward loading (Fig. 1(a)) caused by extreme loading conditions [1]. When snap-through happens, the load–deflection diagram presents a jump, e.g., from point B to point C, as shown in Fig. 1(b). In some cases, when the load is reduced, *hysteresis* is observed: the structure will snap-back to its original configuration but on a different path (represented by the jump from point D to point A in the same figure).

Snap-through is characterized by large nonlinear deformations, changes in the system stability and large stress reversals, which accelerate fatigue damage. Since no analytical solutions for general systems with snap-through exist, numerical models that can capture these phenomena are needed in order to predict the fatigue life of the structure. Among the numerical techniques, the Finite Element Method (FEM) *provides the most generality and can be applied to systems with arbitrarily complex geometries*. This paper analyzes the performance of several finite element formulations (2D and 3D) and the stability of the time-stepping schemes in simulating a curved beam undergoing snap-through: we identify the important features that affect the numerical accuracy and robustness and the region where the schemes are stable for such simulations.

The snap-through of curved beams or shallow arches has been studied analytically for particular cases by numerous authors, including Murphy et al. [2], Virgin [3], Bradford et al. [4], and Plaut and Virgin [5]. Snap-through can also be studied qualitatively using a truss system [6]. Simplifying assumptions are usually needed in finding the analytical estimates. While we are paying the price of the lack of physical intuition by using the complex FEM, we often need to do so in order to use more general analysis techniques that allow for complex effects to be captured. Therefore, we test here the performance of this method when applied to a simple curved beam structure (geometry and properties introduced in Section 2). This structure is representative since it exhibits the type of phenomena we wish to study (snap-through) but simple enough to allow for comparisons with solutions obtained with analytical methods as well.

In this paper, the numerical simulations are performed with the Finite Element Analysis Program (FEAP), a research code that includes most commonly used finite elements and solvers and provides a reliable framework for developing and implementing new user formulations [7]. The factors taken into consideration in this study are (1) the time-stepping algorithm, (2) the element type, and (3) the size of the time step. The element formulations and the time-stepping algorithms discussed are either available from the standard FEAP distribution or implemented as user subroutines. The main criterion used to verify the robustness of the results is the energy conservation throughout the simulation.

For discretization, 2D straight beam elements and 3D solid linear and quadratic elements are used. All formulations account

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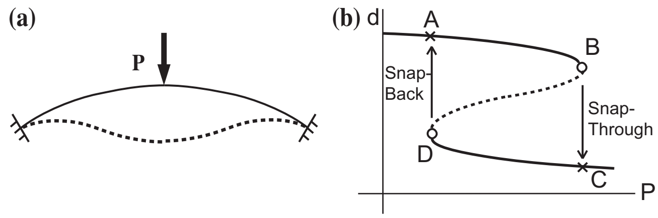

Fig. 1. Snap-through buckling of shallow structures: (a) initial and post-snap configuration, (b) load–deflection curve.

for large deformations. The 2D beam elements utilized are (1) beam elements without shear deformation with *large displacement* and small rotation (2nd order theory) with cubic interpolation, and (2) beam elements with shear deformation formulation and *large displacement and large rotation* that can consider inelastic behavior for bending and axial effects but retain linear elastic response in the transverse shear terms [7]. The 3D solid elements used are linear (8-nodes) elements with displacement, mixed (B-bar), and enhanced formulations and quadratic (27-nodes) elements with displacement formulation. Unlike the *structural* elements, e.g., beams, the continuum (3D solid) elements do not include a built in kinematic assumption. *This characteristic makes them suitable for consistent incorporation of other effects, such as thermomechanical coupling in future studies.* Note that in the current study, no coupling, and no material or boundary nonlinearities are included.

Conventional approaches in the stability analysis of structures often make use of static considerations, but in fact, even when the loads are applied statically, buckling and snap-through are inherently dynamic processes and a full description of the structural behavior can be obtained only through a dynamic analysis [3]. Numerical simulations of such phenomena require access to stable time-stepping schemes and in general to robust simulation environments.

Unfortunately, due to the kinematic assumptions incorporated in the structural elements, the use of these elements coupled with the time-stepping integrators is prone to numerical difficulties that affect the accuracy of the results as shown in Section 4. Numerical instabilities mask the true physical behavior rendering the structural response prediction inaccurate.

An important component of the simulation environment is the time-stepping scheme. We examine here the common choices for structural mechanics simulations: (1) the traditional Newmark integrator [8] and (2) energy–momentum conserving algorithms. The Newmark method is the most widely used time integrator in the area of structural analysis. For certain combinations of parameters, it is unconditionally stable for linear problems. However, its stability is not guaranteed for nonlinear problems. The traditional time-stepping algorithms developed for structural dynamics applications usually perform well for linear problems. In the nonlinear regime, however, numerical instabilities appear due to the energy increase of the discrete system. Hence, energy–momentum schemes were developed to overcome the lack of conservation [9]. These schemes belong to a class of algorithms designed to satisfy various conservation laws by construction. The energy–momentum algorithms used are (1) the algorithm based on the work by Simo and Tarnow [10], Simo et al. [11], and Gonzalez [12] and (2) a composite algorithm based on the trapezoidal rule and the three point backward Euler proposed by Bathe [13] that is stable for large time steps. These algorithms will be referred as conserving A and conserving B, respectively, in the rest of this paper. Many authors used conserving A to solve various types of problems [14]. In a recent work, Garikipati et al. [15] used this algorithm to model growth in biological tissue. Bathe [13] showed that the conserving B algorithm, which is incorporated in the commercial FEM software ADINA 8.7, is able to solve a specific type of problem where the Newmark algorithm is unstable and does not conserve energy and momentum. Although the conserving B scheme allows us to use larger time steps, interactions between the time-stepping schemes and the spatial discretizations might still occur when structural elements are used.

This paper is organized as follows. Section 2 describes the representative structure used throughout the paper. Section 3 compares the results of the static analyses obtained using different element types and formulations. Section 4 analyzes the numerical results of the nonlinear dynamic simulations of a curved beam undergoing snap-through. Various time integrators and element types and formulations are used. A summary of our findings is presented in the concluding section.

2. Representative structure

The representative system discussed in this paper is a curved beam that undergoes snap-through under a concentrated load. The geometry of the beam is shown in Fig. 2. The beam is symmetrical with an angle $\theta = 5.674°$ at the supports. The thicker lines in the figure represent the parts of the beam that are clamped; their horizontal length is L_c. The free portion of the beam has a horizontal projection L_h. The total horizontal projection of the beam, including the clamped parts is $L = 362.204$ mm. The beam can be considered as a thin beam and has a rectangular cross section with depth d much larger than the thickness t. Fig. 3 shows the 3D representation of the beam.

The dimensions of the beam are listed in Table 1 and Table 2 lists the material properties of the structure. Without loss of generality, the type of load used throughout this paper is a point load applied at the center of the beam.

3. Static analysis

In this section we study the effect of the type of elements and formulations on the accuracy of the results in quasistatic simulations. Based on these preliminary results, the options that are less accurate in the static analysis are eliminated, and as a result a decision regarding the best elements to be used in the dynamic analysis can be made.

All simulations presented later in this paper are performed with meshes that ensure spatial convergence. Fig. 4 shows a mesh

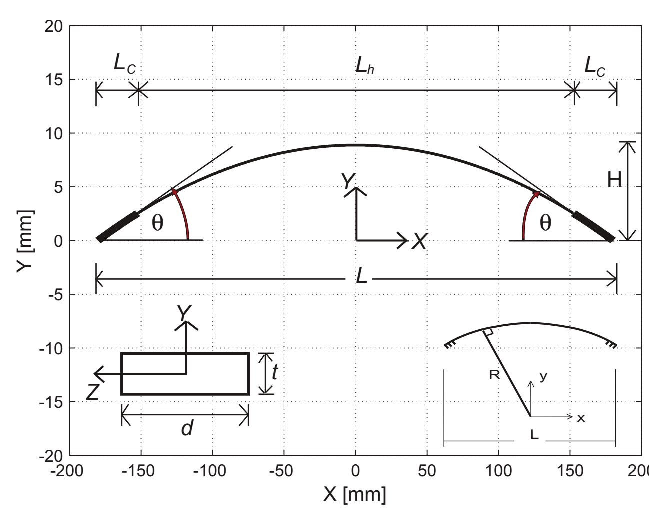

Fig. 2. Geometry of the structure. A curved beam clamped at the supports.