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Lateral-torsional buckling of vertically layered composite beams with interlayer slip under uniform moment

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ABSTRACT

The lateral-torsional stability of vertically layered composite beams with interlayer slip is investigated in this paper, based on a variational approach. Vertically layered elements are typically used in timber engineering but also in case of laminated glass elements. Both across-longitudinal or vertical slip due to rotation and longitudinal or horizontal slip due to lateral deflection are discussed. The theoretical framework of the lateral-torsional buckling problem is given, and some engineering closed-form solutions are presented for partially composite beams under uniform bending moment. Simplified kinematical relationships neglecting the axial and vertical displacements of the sub-elements give unrealistic values for the lateral-torsional buckling moment. Refined kinematical assumptions remove this peculiarity and render sound buckling moment results. Inclusion of the horizontal and vertical slips significantly affect the lateral-torsional buckling moment of these vertically laminated elements. A single lateral-torsional buckling formulae is derived, depending on both the horizontal and the vertical connection parameters.

1. Introduction

Composite structures of different materials have important applications in civil or mechanical engineering. The lateral-torsional stability of composite beams with partial interaction is investigated in this paper, based on a variational approach. As known to the authors, no earlier studies have been devoted to this stability problem for composite beams with interlayer slip. The basic theoretical framework of lateral-torsional buckling of partially composite beams is discussed, and some simplified engineering results are presented. Composite beams with interlayer slip studied in this paper represent a number of different classes of structures with the same type of basic behaviour and governing equations, such as layered beams (mechanically jointed), shear connections (lap joints mechanically or adhesively jointed) and sandwich constructions (with weak shear cores). This similarity is also demonstrated in this paper. Also, there exist different types of buildings where part or the whole of the structure is modelled as a partially composite or sandwich-type of structure.

Layered structural elements with interlayer slip are typically encountered in wood design, where wooden beams are made up of layers assembled by means of nailing, bolting or gluing (with a soft shear modulus). Partially composite structures built up by sub-elements of different materials and connected by shear connectors to form an interacting unit, such as timber-concrete or steel–concrete elements, are widely used in building engineering. In the case of a flexible connection, the analysis procedure requires consideration of the interlayer slip between the sub-elements, leading to the partial interaction concept. For a detailed literature background on the partial composite theory, the readers are referred to Girhammar and Gopu [27] and Girhammar and Pan [28]. With respect to the original developments of the fundamental differential equations the work of especially [31] and Newmark et al. [52] should be mentioned. A central work summarizing these early theories is that of Goodman and Popov [29]. Rassam and Goodman [57] generalized the results obtained by Möhler [51] for the buckling formulae of axially loaded partially-composite columns. The lateral buckling problem of partially composite beam-columns subjected to both transverse and axial loading was investigated by e.g. Girhammar and Gopu [26], Girhammar and Gopu [27] and Girhammar and Pan [28] who used Euler–Bernoulli beam models for each beam-column. Based on linear behaviour of the materials and shear connections, this in-plane buckling problem is ruled by a 6th order differential equation for the deflection and the buckling load can be obtained for general boundary conditions. Cas et al. [12] also analysed

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this in-plane buckling problem, but by using a finite element procedure and for a more complex constitutive formulation. Xu and Wu [63] generalised the results of Girhammar and Gopu [27] by using Timoshenko's beam theory. Buckling of composite beam-columns with interlayer slip and differential shear model has been recently studied by Schnabl and Planinc [60] and Challamel and Girhammar [21].

Sandwich beams are usually composed of three layers, two thin faces and a thick weak shear core [53,54,1]. These sandwich beams have been widely studied in the 1960's especially for their damping dynamic characteristics (e.g. [43,23,49]. The static in-plane behaviour of sandwich beams is well established by Hoff [40] (see e.g. also [32]. A recent application of these three-layer sandwich beams is the design of laminated glass beams [2,42,11,25,46,47]. Laminated glass comprises two glass layers bonded together by an elastomeric polymer called polyvinyl butyral (with soft modulus). Laminated glasses can be used in contemporary buildings as architectural glazing. Another application in building engineering is found for high-rise buildings, which can be modelled as equivalent sandwich beams, see e.g. Potzta and Kollár [55]. The in-plane buckling problem of a three-layer sandwich beam was studied by Hoff and Mautner [39] (see e.g. also [40,8,44]. A 6th order differential equation is obtained for the deflection, and the similarity between this governing equation and that for composite beams with partial interaction was demonstrated by Heuer [36]. However, it should be mentioned that the equivalence between the slip modulus of shear connectors and shear modulus of glue lines of finite thickness or sandwich-type of cores was first demonstrated by McCutcheon [48] (see also [20]. The buckling formula of Hoff and Mautner [39] for sandwich beams cannot be cast in the Engesser ́s or Haringx's formula [24,33]. For discussion of the relevancy of Engesser and Haringx respective theory for sandwich columns, the reader is referred to Bažant [9] (see also [61,10,4]. Along with the work of Hoff and Mautner [39] with respect to the generic formula of buckling loads of sandwich columns, the theoretical works of Hegedüs and Kollár [34,35] should be mentioned (see also [5] or [45].

The lateral-torsional buckling behaviour of prismatic homogeneous beams has been extensively investigated since the pioneering studies of Michell [50] and Prandtl [56], more than one century ago. However, exact analytical results for the lateral-torsional buckling of homogeneous beams, even in the simplified version of slender beams, are very scarce (see e.g. [62,7]). For instance, Challamel [13] shows the difficulty to derive exact buckling formula in case of multi-dimensional loading composed of coupled distributed and concentrated forces. Using Bessel's functions Challamel and Wang [16] obtained the exact stability domain for the lateral-torsional buckling problem of a cantilever beam solicited by two concentrated forces. Theoretical solutions based on confluent hyper-geometric functions have recently been obtained for very specific tapered beams [14]; see more recently [17]. The lateral-torsional buckling of nonlocal beams is also recently considered by Challamel and wang [19] with possible applications to nanotechnology. Some lateral-torsional buckling solutions, including the shear effect that can be predominant for (fibre reinforced plastic composite) beams, have been derived ([58,45]). Sapkás and Kollár [59] obtained some closed-form solutions for the lateral-torsional buckling moment of (pultruted fibre reinforced plastic composite) beams taking both the lateral and torsional shear deformation into account (expressed as Engesser type of formulae). Recently, Attard and Kim [6] rigorously derived a spatial stability theory of beams that account for shear effects, leading to meaningful lateral-torsional buckling solutions. Attard and Kim [6] show that the shear effect introduced from Reissner's theory has no influence on the lateral-torsional buckling moment in case of uniform moment (hinged–hinged boundary conditions). Of course, as shown by Reissner [58], shear effect can be significant in case of cantilever beams loaded by concentrated forces. With respect to the composite beam considered in the paper, the lateral-torsional buckling problem of glued laminated beams is investigated by Hooley and Madsen [41], who treated the glued laminated beam as a solid homogeneous beam.

To the authorś knowledge no closed-form solutions have been presented concerning lateral-torsional buckling of composite beams with partial interaction or sandwich-type of beams. However, it can for the sake of completeness be mentioned that Dall'Asta [22] developed a three-dimensional theory for composite beams with partial interaction (weak shear connection) subjected to combined torsion and transverse bending (in the strong axis). Challamel [15] or Challamel et al. [18] investigated the free vibrations analysis of the out-of-plane behaviour of composite beams with interlayer slip. Amadio and Bedon [3] recently investigated the lateral-torsional buckling of vertically layered beams. Amadio and Bedon [3] only considered the in-plane slip between the two subelements. We suggest in this paper to generalize their results with the introduction of the out-of-plane slip coupling. There is a definite need to start establishing an analytical procedure for the analysis of lateral-torsional buckling of composite members with partial interaction and working out the lateral-torsional buckling loads for different engineering applications. It is seems appropriate to start with simple loading and boundary conditions and, also, using some simplifying assumptions.

2. General analysis model

The geometric parameters defining a typical composite beam built up of two vertically connected sub-elements of different geometry and materials are shown in Fig. 1a. The subscripts '1' and '2' refer to each sub-element of the cross-section, respectively. The x-axis of the coordinate system is located in the shear center (SC). The displacements in the x-, y-, and z-direction are denoted u, v, and w, respectively.

The possible slip (Δv)and slip forces ($V_{s,y}$) in the across-longitudinal or horizontal direction due to rotation of the cross-section are illustrated in Fig. 1b. A horizontal view of the free-body diagram of an element in the composite beam is shown in Fig. 1c, where moments (M_z, M_{z1}, M_{z2}) and shear forces (V_y, V_{y1}, V_{y2}) are defined. A corresponding vertical view is shown in Fig. 1d, where moments (M_y, M_{y1}, M_{y2}), shear forces (V_z, V_{z1}, V_{z2}), normal forces (N_1, N_2), and slip (Δu) and slip forces per unit length ($V_{s,x}$) due to lateral deflection are defined. For the general case with sub-elements of different geometries and materials, it is assumed that the vertical forces act in the shear centre plane and, hence, no skew bending takes place. The influence of shear forces on the deformations will be neglected in this study.

The sub-elements are connected together by means of some kind of a weak shear layer, which produce uniformly distributed slip forces or interlayer shear stresses with a constant slip modulus per unit length, K_{along} [N/m^2], with respect to bending in the lateral direction (slip in the longitudinal or horizontal direction) and, K_{across} [N/m^2], with respect to torsion (slip in the across-longitudinal or vertical direction). The slip modulus are assumed to be constant along the tangential plane, i.e. both in the x- and in the y-direction. In the general case, the connection is assumed to be orthotropic with different slip moduli in the two directions, which are defined as

$$\begin{bmatrix} V_{s,x} \\ V_{s,y} \end{bmatrix} = \begin{bmatrix} K_{\mathrm{along}} & 0 \\ 0 & K_{\mathrm{across}} \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix} \quad (1)$$

In this paper, different alternatives will be studied, no shear connection in the longitudinal ($K_{\mathrm{along}} = 0$) or in the across-longitudinal ($K_{\mathrm{across}} = 0$) direction and equal slip modulus in the two directions, i.e. $K_{\mathrm{along}} = K_{\mathrm{across}} = K$ (isotropic connection law). Frictional effects