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Least-work solutions of flange normal stresses in thin-walled flexural members with high-order polynomial

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ABSTRACT

An energy-based method was developed for quantifying shear lag effects in thin-walled flexural members such as box girders, T-beams, and nonrectangular concrete walls. The proposed procedure uses infinite terms of high-order polynomial to describe the uneven longitudinal displacement in the flanges. The series type of approximation resulted in a group of coupled differential equations, for which solution techniques were developed. The proposed variational analysis was compared with the existing least-work solutions and two experimental tests of rectangular box girders in the literature and one of tests of steel box beams in this study. The comparisons indicated that the proposed variational analysis can accurately predict the flange normal stresses in box girders. Solutions were provided for thin-walled flexural members in bridges and buildings under a variety of loadings and boundary conditions to facilitate the implementation of the proposed procedure.

1. Introduction

The shear lag effect in this paper describes the unevenly distributed normal stress in the flange of a thin-walled flexural member, as shown in Fig. 1. Due to shear lag effects, the structural behavior can be different from that predicted by the elementary beam theory, which assumes that the normal stress in a slender beam is proportional to the distance from the neutral axis. Shear lag has been identified and studied in many engineering structures, including airplane structures [1–3], high-rise buildings [4–6], composite beams [7–12], and box girder bridges [13–15]. Two concepts, effective flange width [10–12,16,17] and stress increase factor [9,18,19], are widely used in engineering design practices. Effective flange width is the partial flange in tension/compression, with which the largest normal stress in a thin-walled flexural member can be obtained following the elementary beam theory. The stress increase factor modifies the normal stress calculated using the original cross section and the elementary beam theory such that the peak normal stress in the flange can be obtained for design. The determination of both design parameters requires an accurate normal stress distribution across the flanges of thin-walled flexural members.

The existing shear lag analysis techniques include analytical procedures (e.g., the finite stringer method [20,21], the biharmonic analysis [1,9,22], and energy-based analyses [2,6,14, 18,23,24]), numerical analyses [10–13,17,19], and experimental tests [18,25]. The results of energy-based analyses have been used in the design of box girder bridges and high-rise buildings [5,6,14, 26]. In the existing energy-based analysis, the longitudinal flange displacement is described using a quadratic or cubic polynomial term with one unknown parameter. The variational principle is applied to the potential energy of the member to determine the unknown parameter. The accuracy of the energy-based analyses is thus limited by the ability of the assumed polynomial to approach the longitudinal displacement across the flanges, which varies along the member as observed in several studies [19,27].

A $2m$-degree polynomial (the summation of m terms of binomials with even exponents) was used in this study to approach the actual longitudinal displacement in the flanges of thin-walled flexural members with various boundary conditions and loadings. The proposed series type of approximation resulted in a group of coupled differential equations, for which solution techniques were developed by solving an eigenvalue problem. This procedure is described below following a review of the existing analysis techniques. The analyses of two steel box girders tested and documented in the literature and a steel box beam in this study were used to demonstrate the effectiveness of the proposed procedures.

2. Literature review

Analytical procedures are needed to provide guidance for experimental tests and numerical studies. Among the existing

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Notation

A_f, A_w, A Area of the flange, the web and the cross section of a box girder
A_s Area of stiffeners on the compression flange
b_1, b_2, b_3 Half width of bottom flange and top flange, and overhanging flange width
b_e Effective flange width
b_l Width of the distributed load equivalent to the applied point load
d_j, e_j Coefficients in the matrix $\boldsymbol{U}_h$
f_1 Geometrical parameter
E, G Young's modulus, shear modulus
h, L Height and the span of the box girder
I_s, I_w, I Moment of inertia of the flanges, the webs and the box girder
n, k Two parameters defined by Reissner
M, V Moment and shear in the flexural member
M' Derivative of the moment with respect to x
P, a Point load and the position of the load
Q_w First moment of the web
q, q_o Uniformly distributed load, and the peak of the triangular loads
t_1, t_2, t_3, t_w Thickness of bottom flange, top flange, cantilever flange, and web plate
$u(x, y)$ Normal flange displacement of a box girder
$U_i(x), U_i'(x)$ Parameter for the ith term and its first derivative in the polynomial function
$\boldsymbol{U}_h, \boldsymbol{U}_p$ Homogeneous solution and particular solution of unknown parameters
ψ Effective flange width ratio ($\psi = b_e/b$)
α_s Timoshenko shear coefficient
σ_x, ε_x Normal stress and normal strain of a box girder
ν Poisson's ratio of girder material
$\varphi(x), \varphi'(x)$ Curvature of the girder and the first derivative of the curvature
λ_i, Φ_i Eigenvalue and the eigenvector for the ith term of $U_i(x)$
$\gamma_{xy}, \tau_{xy}, \tau_{xz}$ Shear strain and shear stress in xy-plane, shear stress in xz-plane

analytical procedures, stress in the flange is obtained by solving the partial differential equation using the method of separation of variables and applying the boundary conditions at member ends. For example, the flange normal stress (σ_x) for a simply supported box girder subjected to a point load as shown in Fig. 2 is,

$$\sigma_x = \sum_{k=1}^{\infty} \frac{Pbh \sin\lambda a \sin\lambda b_l \sin\lambda x}{Ib_l\lambda^2 L\,(2b\lambda + \sinh 2\lambda b)}\,[-2\lambda b \sinh\lambda b \cosh\lambda y + 2\cosh\lambda b(2\cosh\lambda y + y\lambda \sinh\lambda y)]\,, \quad (1)$$

where $\lambda = k\pi/L$ for simply supported girders, h is the distance from the center of the flange to the neutral axis, b is the half flange width, and b_l is the width of a patch load that is equivalent to the point load P placed at $x = a$ from the left support.

The force boundary conditions along the flange sides are determined by the equilibrium of the flange plate: the shear stress (τ_{yx}) is assumed to balance the normal stresses across the flange (σ_x). This normal stress is usually obtained using the elementary beam theory, leading to a mistaken presumption that the resultant of the normal stress across the flange is the same as those obtained without considering shear lag effects. With the presumption, the shear stress (τ_{yx}) along the flange sides takes the same shape as the shear diagram, which is then approximated using a Fourier series

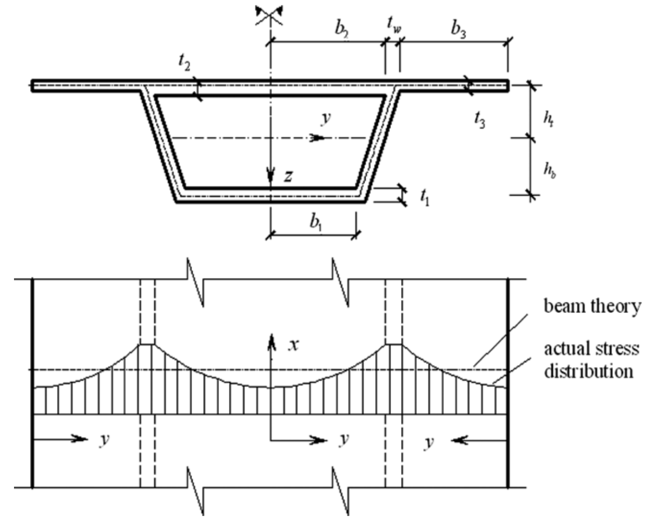

Fig. 1. Schematics of shear lag effects in a box girder.

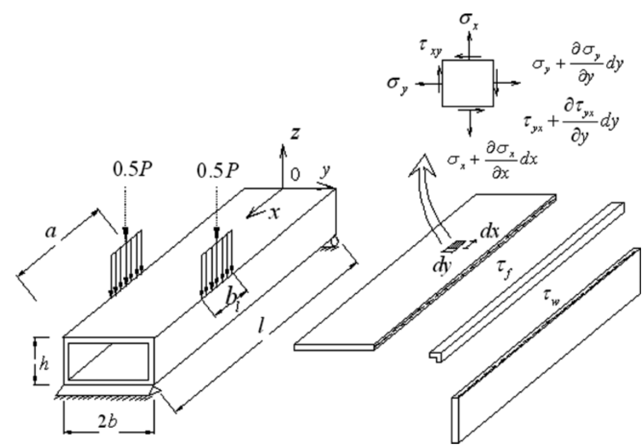

Fig. 2. Bi-harmonic analysis of isolated flange plate.

before being used in the solution of the bi-harmonic equation. A Fourier series converges with sufficient accuracy for girders under distributed loads. However, it is difficult for a Fourier series to accurately approach the shear diagram of a girder with a point load, where a discontinued point exists at the position of the load. The discontinuity causes Gibbs phenomenon [28,29] in the Fourier series near the load point. Although several methods are available to reduce the Gibbs effect (e.g., Fejer summation [28] and sigma-approximation [29]) in a truncated Fourier series, excessive number of terms are needed in the summation of the Fourier series to obtain accuracy stable solution. Furthermore, a convergence problem exists in the solution near a point load; hence the point load is usually approached with a short distributed load [1,9,22]

Unlike bi-harmonic analysis, which ignores member geometry and material properties as shown in Eq. (1), the energy-based method establishes equations using the potential energy of the entire member. The necessary assumption is that shear lag effects on longitudinal flange displacement, $u(x, y)$, can be described using a second-order (or third-order) binomial,

$$u(x, y) = \pm h\left[w'(x) + \left(1 - \frac{y^2}{b^2}\right)U(x)\right], \quad (2)$$

where, $w'(x)$ is the derivative of the beam deflection, h is the distance of the center of the flange to the neutral axis, which has been usually set at the middle height for a rectangular box section, b is the half flange width, and $U(x)$ is the unknown parameter.