Engineering Structures 33 (2011) 2762–2771

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Nonlinear dynamic behavior of saddle-form cable nets under uniform harmonic load

Isabella Vassilopoulou [a,*], Charis J. Gantes [b,1]

[a] Laboratory of Metal Structures, School of Civil Engineering, National Technical University of Athens, 12, Irinis Avenue, GR-15121 Pefki, Greece
[b] Laboratory of Metal Structures, School of Civil Engineering, National Technical University of Athens, 9 Heroon Polytechniou Street, GR-15780 Zografou, Athens, Greece

ARTICLE INFO

Article history:
Received 2 March 2011
Received in revised form
17 May 2011
Accepted 1 June 2011
Available online 2 July 2011

Keywords:
Saddle-form cable net
Nonlinear dynamic response
Superharmonic resonance
Jump phenomena

ABSTRACT

The dynamic response of saddle-form cable nets is investigated in this paper. Even though they consist of cables, which are well known for their geometric nonlinearity, such systems could be characterized as weakly nonlinear due to the high levels of pretensioning of their cables and to their hyperbolic paraboloid surface, having opposite curvatures at all points and thus increased stiffness. Nevertheless, resonance phenomena that are typical of highly nonlinear systems are detected here, for common geometries and levels of pretension, even for low levels of load amplitude. First, a single-degree-of-freedom (SDOF) cable net is studied analytically and numerically, and nonlinear resonances are confirmed. Then, the response of multi-degree-of-freedom (MDOF) cable nets, subjected to harmonic dynamic excitation, is investigated. Although the static response is proved to be almost linear, the dynamic nonlinearity is intense, as verified by jump phenomena, bending of the response curve, superharmonic resonances, and dependence on the initial conditions.

1. Introduction

Cable nets belong to the family of tensile structures that are characterized by their capacity to carry loads much heavier than their own weight. The most common shape of cable nets is the hyperbolic paraboloid. The net consists of two families of cables: the carrying or main cables that produce the concave surface, which are anchored at the highest points of the boundary, and the stabilizing or secondary ones, which, anchored at the lowest points of the boundary, create the convex surface. Cable structures differ from conventional linear systems, due to their nonlinear response to both static and dynamic actions. Their response cannot be obtained on the basis of their original undeformed geometry, because their stiffness increases as the deflection increases, and the internal forces do not vary linearly with load. Therefore, it is necessary to take into consideration the deformed state at every step of the load, performing nonlinear analyses, which account for large displacements.

The dynamic response of a nonlinear system is unpredictable, as several nonlinear phenomena may appear, such as secondary resonances, including superharmonic and subharmonic resonances, depending on the relation between the loading frequency Ω and the eigenfrequencies ω_i of the system. If $\Omega \approx n \cdot \omega_i$ or $\Omega \approx (1/n) \cdot \omega_i$, where n is an integer, subharmonic or superharmonic resonance may occur, respectively. In such cases, all modes involved in the secondary resonances are activated during the oscillation [1,2]. The relation between oscillation amplitude and frequency can be described by a response diagram, in which the steady-state amplitude is plotted on the vertical axis and the frequency ratio Ω/ω on the horizontal axis, where Ω is the loading frequency and ω the natural frequency of the system. For a free vibration of an undamped oscillator, the steady-state response is described by one line, known as the backbone. For a forced system, the steady-state response is represented by different curves, depending on the amplitude of the external force. These curves can be interpreted as perturbations out of the equilibrium state. In linear systems, the backbone is a straight vertical line and the response curves for the forced systems approach this line asymptotically, as the forcing frequency Ω approaches the system's frequency ω, indicating the phenomenon of fundamental resonance, in which the vibration amplitude increases infinitely when the force has the same frequency as the system. In nonlinear systems instead, the backbone is a bending curve accounting for either the softening or the hardening behavior of the system (Fig. 1). The softening behavior means that the stiffness of the system decreases as the oscillation amplitude increases, while in the hardening behavior the system's stiffness becomes progressively higher for large amplitudes [3].

Several studies have been published in the past regarding nonlinear dynamic phenomena for individual cables [4–6], while some

* Corresponding author. Tel.: +30 210 6141055; fax: +30 210 7723442.
E-mail addresses: isabella@central.ntua.gr (I. Vassilopoulou), chgantes@central.ntua.gr (C.J. Gantes).
1 Tel.: +30 210 7723440; fax: +30 210 7723442.

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doi:10.1016/j.engstruct.2011.06.001

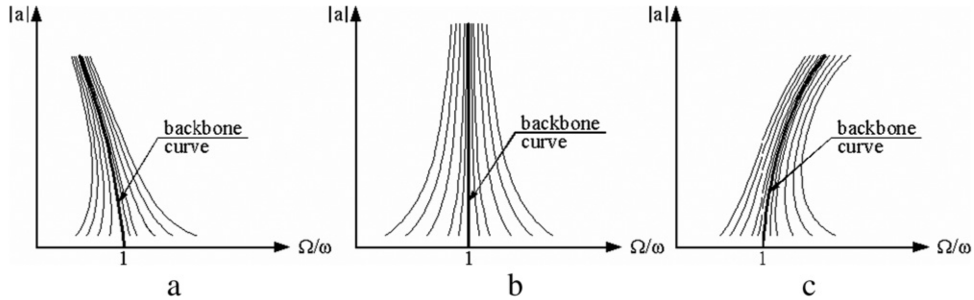

Fig. 1. Amplitude–frequency curves for undamped systems with (a) softening nonlinear behavior, (b) linear behavior, and (c) hardening nonlinear behavior.

recent findings and experimental results are included in [7,8]. However, only a few investigators have dealt with such phenomena for cable net structures. Most publications referring to cable nets have presented computerized methods of analysis and other numerical techniques to calculate the nonlinear static or dynamic response of cable networks and membranes by solving the governing equations of motion [9–13]. In [14], the two most common methods of analysis for cable roof structures subjected to wind actions were proposed to be used, which are the frequency domain analysis and the time domain analysis. The former method, commonly used for linear structures, can be applied for approximating the response of weakly nonlinear structures. For strongly nonlinear structures, time domain analyses, taking into account the geometrical nonlinearity, were recommended as the only appropriate method.

During the initial stages of the present work, the dynamic behavior of an undamped cable net with fixed supports under fundamental resonance was explored in [15], and internal resonances were detected, indicated by the beat phenomena in the oscillation of the net. On the other hand, in [16], the dynamic behavior of a damped saddle-form cable net with rigid supports subjected to a uniformly distributed load was analyzed for a wide range of the loading frequency, concluding that it is never sufficient to take into consideration only the first natural modes, as fundamental resonances of higher modes may lead to cable net oscillations of large amplitudes, comparable to those generated by the fundamental resonance of the first symmetric mode.

In the present work, first, the analytical equations of motion are given for the simplest cable net, consisting of two crossing cables. The central node is subjected to a harmonic load, with varying frequency. Bending of the response curve, double responses dependent on the initial conditions, jump, and superharmonic resonances are detected. Then, a multi-degree-of-freedom (MDOF) cable net is considered, with fixed cable ends, taking as an example the cable net roof of the Peace and Friendship Stadium in Athens, Greece [17], shown in Fig. 2. The dynamic response of the cable net, subjected to harmonic excitations with a uniform spatial distribution, is investigated. As analytical solutions turned out to be practically impossible for full-scale three-dimensional cable structures due to their complex nonlinearity, numerical analyses are performed, using finite element software, which is validated for the simple cable net.

This investigation aims at evaluating the nonlinear nature of cable nets, through the occurrence of nonlinear dynamic phenomena, in order to assess if they can be treated as weakly nonlinear systems and if they can be reliably analyzed by linear quasi-static procedures.

2. Equations of motion of a simple model of a cable net

A simple structure of two crossing systems of prestressed pin-ended bars (not undergoing compression) is assumed (Fig. 3), which is referred to in the following as a system of two crossing cables, having equal spans L and sags f, and the same cross sectional area A, made of linear elastic material in tension, with Young modulus E and initial elongation ε_0, which is interpreted as initial pretension T_0, according to Hooke's law:

Fig. 2. The stadium of Peace and Friendship in Athens under construction.

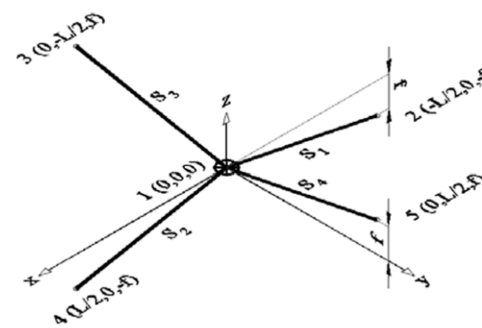

Fig. 3. Geometry of the simple cable net with two cables.

$$T_0 = EA\varepsilon_0. \tag{1}$$

The material is assumed to have zero compression branch. A concentrated mass is applied on the central node. The initial prestressed length of each segment is

$$S_N = \sqrt{(L/2)^2 + f^2}. \tag{2}$$

If the displacements of the central node, referring to the global axes x, y, z, are defined as w_x, w_y, w_z, respectively, the deformed lengths of the cable segments will be

$$\begin{aligned} S_{1,2} &= \sqrt{(L/2 \pm w_x)^2 + w_y^2 + (f + w_z)^2} \\ S_{3,4} &= \sqrt{w_x^2 + \left(L/2 \pm w_y\right)^2 + (f - w_z)^2}. \end{aligned} \tag{3}$$

According to Hooke's law, and assuming small strains, the cable tension for each segment will be

$$\begin{aligned} T_i &= E \cdot A \cdot (S_i - S_0)/S_0 \\ &= T_0 + (E \cdot A \cdot (S_i - S_N)/S_0), \quad i = 1, 2, 3, 4. \end{aligned} \tag{4}$$

S_0 is the initial unstressed length for all segments, equal to

$$S_0 = \frac{S_N}{1 + \varepsilon_0}. \tag{5}$$