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Response variability of cylindrical shells with stochastic non-Gaussian material and geometric properties

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ABSTRACT

In this paper, the effect of combined uncertain material (Young's modulus, Poisson's ratio) and geometric (thickness) properties on the response variability of cylindrical shells is investigated taking into account various non-Gaussian assumptions for the uncertain parameters. These parameters are described by two-dimensional univariate homogeneous non-Gaussian stochastic fields using the spectral representation method in conjunction with translation field theory. The response variability is computed by means of direct Monte Carlo simulation (MCS). It is shown that the marginal probability distribution and the correlation scale of the stochastic fields used for the description of the material and thickness variability affect significantly the shell response statistics.

1. Introduction

A powerful tool in computational stochastic mechanics is the stochastic finite element method (SFEM). SFEM is an extension of the classical deterministic FE approach to the stochastic framework i.e. to the solution of stochastic problems whose (material and geometric) properties are random with the FE method. The considerable attention that SFEM received over the last two decades can be mainly attributed to the understanding of the significant influence of the inherent uncertainties on systems' behavior and to the dramatic increase of computational power in recent years, permitting the efficient treatment of complex realistic problems with uncertainties [1,2].

A characteristic example of structures with a complex stochastic response is that of shell structures. The analysis and design of shells are challenging since their behavior can be unpredictable with regard to geometry or support conditions. In particular, the extreme sensitivity of thin shells to imperfections in material, geometry and boundary conditions requires a realistic description of all uncertainties involved in the problem. This task is realizable only in the framework of a robust SFEM formulation that can accurately and efficiently handle material and geometric uncertainties. The need for a robust, accurate and computationally efficient shell element becomes even greater for the computationally expensive SFE analysis of large realistic shell structures.

The TRIC (TRIangular Composite) shear-deformable facet shell element is a reliable and cost-effective triangular element suitable for the linear and nonlinear analysis of thin and moderately thick isotropic as well as composite plate and shell structures [3]. Its formulation is based on the natural mode finite element method, which has a number of computational advantages compared to the conventional isoparametric finite element formulations. The treatment of the element kinematics (inclusion of the transverse shear deformations in its formulation based on a first order shear-deformable beam theory) eliminates locking phenomena in a physical manner. The rigorous theoretical basis of the element has been confirmed in several publications, while numerical examples have verified its accuracy and computational efficiency in various structural applications [3–5]. An important feature of the TRIC element in the context of stochastic analysis is the fact that there is no need to perform numerical integration for the computation of its deterministic stiffness matrix, which is carried out in closed form. This special feature of the TRIC element provides an ideal basis for the formulation of a computationally efficient stochastic stiffness matrix and for the use of the element in large-scale stochastic computations.

In most SFEM applications, a straightforward randomization of only one material property is performed by assuming that this property is described by a stochastic field, e.g. [6–10]. For example, the Young's modulus is often assumed to vary randomly over space, while the Poisson's ratio is considered as a deterministic

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constant. However, many of the physical mechanisms that lead to random variations of Young's modulus also lead to random variations in other material properties such as Poisson's ratio. A formulation for the SFE analysis of plate structures with an uncertain Poisson's ratio has been proposed in [10]. This formulation is based on a decomposition of the constitutive matrix into several sub-matrices via polynomial expansion of the Poisson's ratio and is combined with a first order Taylor expansion for the calculation of the response statistics. An alternative SFE approach accounting for secondary effects due to material randomness has been introduced in [11]. In this approach, stochastic shape functions are computed based on local equilibrium criteria and the stochastic stiffness matrix is calculated using the corresponding stochastic strain–displacement matrix. High accuracy is achieved with this SFE technique, which is preserved in the case of a large stochastic variation of the input parameters. The case of multiple uncertain material and/or geometric properties represented by random variables or random fields is treated in a few publications [12–19].

In all the aforementioned publications, a Gaussian assumption is made for the random variables or random fields representing the uncertain parameters of the problem. However, the use of the normal distribution for material and geometric properties bounded by physical constraints is questionable especially in the case of large coefficients of variation as there is a non-zero probability that a violation of these constraints might occur. Moreover, it has been shown in [20] that the variance of the response of a system with Gaussian stiffness is infinite. Therefore, a non-Gaussian assumption is more appropriate for a physically sound and accurate description of material and geometric properties [21].

In this paper, the effect of combined uncertain material and geometric properties on the response variability of a thin cylindrical shell is investigated taking into account various non-Gaussian assumptions for the uncertain parameters. To this purpose, a non-Gaussian spatial variability of the Young's modulus and Poisson's ratio as well as of the thickness of the shell is considered. These parameters are described by two-dimensional univariate (2D-1V) homogeneous non-Gaussian stochastic fields using the spectral representation method in conjunction with translation field theory [22,23]. The stochastic stiffness matrix of the TRIC shell element is based on the local average [24] and weighted integral [6] methods and depends on a minimum number of random variables representing the stochastic fields.

The numerical example focuses on the influence of the non-Gaussian assumption on the response variability of the shell structure, which is quantified in terms of exceedance probabilities by means of direct Monte Carlo simulation (MCS). The influence of the variation of each random parameter, as well as of the correlation scale of the stochastic fields, is also investigated. It is shown that the marginal probability distribution used for the description of the material and thickness variability affects significantly the response statistics of the shell.

2. Stochastic finite element formulation

The stochastic finite element analysis is performed using the multi-layered triangular shell element TRIC, the formulation of which is based on the natural mode method. A schematic representation of the element appears in Fig. 1. The element has 18 degrees of freedom (6 per node) and hence 12 natural straining modes: 3 axial straining modes, 3 symmetric bending modes, 3 anti-symmetric bending + shear modes and 3 in-plane rotations. Its natural stiffness is based only on deformations and not on associated rigid-body motions. The 12 × 12 stiffness matrix k_N corresponding to the natural modes is denoted as the natural stiffness matrix of the element. A detailed description of the formulation of the deterministic linear elastic stiffness matrix of the TRIC shell element can be found in [3].

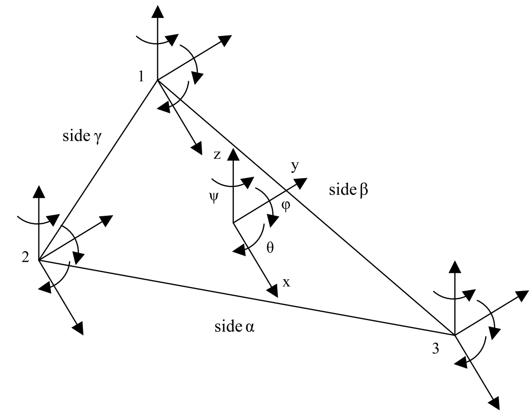

Fig. 1. The multi-layered triangular shell element TRIC.

2.1. Random variation of the Young's modulus and Poisson's ratio

As the elements of the stiffness matrix are nonlinear functions of Poisson's ratio ν, the analysis of shell structures involving both the stochastic Young's modulus and Poisson's ratio requires the introduction of additional approximations. An alternative weighted integral formulation of the stochastic stiffness matrix is possible in this case, considering Lamé's constants λ and μ as the two uncertain material properties since all elements of the stiffness matrix are linear functions of λ and μ [13].

For the TRIC shell element and in the case of isotropic material, the elasticity matrix κ_{12} in the material coordinate system can be expressed as

$$\kappa_{12} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix}. \tag{1}$$

The Lamé's constants are defined as

$$\lambda = \frac{\nu E}{1-\nu^2}, \qquad \mu = G = \frac{E}{2(1+\nu)}. \tag{2}$$

After some algebra, Eq. (1) becomes

$$\begin{aligned}\kappa_{12} &= \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & 2\mu \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\kappa_{12}(\lambda)} + \underbrace{\begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & 2\mu \end{bmatrix}}_{\kappa_{12}(\mu)} \\ &= \kappa_{12}^{(1)} + \kappa_{12}^{(2)}. \end{aligned} \tag{3}$$

The two material properties are assumed to vary randomly along the element surface Ω according to

$$\lambda(x,y) = \lambda_0[1 + f_\lambda(x,y)] \tag{4a}$$

$$\mu(x,y) = \mu_0[1 + f_\mu(x,y)] \tag{4b}$$

where λ_0, μ_0 are the mean values of Lamé's constants and $f_\lambda(x,y)$, $f_\mu(x,y)$ are 2D-1V zero-mean homogeneous stochastic fields corresponding to the spatial variation of λ and μ respectively. In this work, a non-Gaussian assumption is made for f_λ, f_μ as described in Section 3.

Substituting Eqs. (4a), (4b) into the expression of the elasticity matrix κ_{12}, we have

$$\kappa_{12} = [\kappa_{12}^{(1)}]_0[1 + f_\lambda(x,y)] + [\kappa_{12}^{(2)}]_0[1 + f_\mu(x,y)] \tag{5}$$