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Geometric issues and ultimate load capacity of masonry arch bridges from the northwest Iberian Peninsula

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ABSTRACT

This paper reports the results from a geometrical survey carried out on 59 segmental masonry arch bridges from Portugal and Spain, with a focus on the adjacent geographical areas of northern Portugal and northwestern Spain. Historical empirical rules are briefly presented and further compared against the bridges' geometrical data. Departing from a detailed discussion of the geometrical results, eight reference bridges were defined as representative of the sample. Subsequently, the paper deals with a parametric assessment of the load-carrying capacity of the reference bridges and a discussion of results. It was found that arch thickness and physical properties of the fill are of paramount importance in terms of ultimate load-carrying capacity. Furthermore, the results indicate that the bridges from the sample are structurally safe with respect to applicable legislation.

1. Introduction

The use of arches and vaults to span horizontal spaces is several thousand years old. The first arches were found in underground tombs in Mesopotamia,built around 3000 BC [1]. Besides the Sumerians, the Egyptians and Greeks also knew vault and arch structures [2]. Following a long process of evolution, Etruscans are considered the first to have built masonry arches using wedge stones. Later on, Romans not only improved arch construction techniques but also added pozzolanic mortar [2]. With the decline of the Roman Empire, around the 5th century, the road system, including the bridges, suffered a significant degradation. Six centuries later, the occurrence of important economic and social changes in Europe caused an increase in economic activity, requiring the construction of new roads and bridges. The construction of masonry arch bridges returned to Europe with splendour.

The ensemble of European masonry arch bridges is a result of centuries of hard and delicate work, thus representing an invaluable architectural and cultural heritage. Nowadays, it is still possible to find Roman bridges, characterized by their flat pavements and identical semicircular arches, as well as the more flexible mediaeval bridges, with larger central spans, semicircular or pointed arches, cutwaters and humpback pavements. However, the successive maintenance and repair works to which bridges were submitted through the centuries has generally led to difficulties in the dating process, resulting in sometimes erroneous classification [3]. With time, the significant change in load from those for which bridges were initially built, the unavoidable decay of materials and a persistent lack of maintenance have led to varying degrees of damage, with many bridges now not compatible with their current use and some even structurally unsafe.

Following earlier developments (e.g. [4–6]), the last two decades have witnessed very important advances concerning the mechanics of masonry arch bridges. These advances include the development of methods for the computation of load-carrying capacities involving different degrees of complexity (e.g. [7–10]) and of assessment techniques based on non-destructive testing methods (e.g. [11]). The few works carried out on masonry arch bridges in Portugal [12–19] have centred on geometrical and architectural descriptions, surveys of damage and strengthening possibilities, while structural analysis and assessment are almost absent. As a consequence, the structural safety of Portuguese masonry arch bridges is hardly known. This paper encompasses the study of a sample of 59 roadway masonry arch bridges, including geometrical and structural analysis of the sample and comparison with historical empirical rules. The main objectives are the characterization of the geometry of bridges constructed in adjacent geographical areas and the achievement of rapid screening of structural safety of the sample based on geometrical information and relatively simple numerical tools. The research work carried out is based on the following steps:

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Table 1
Historical empirical rules for crown arch thickness.

Date	Author	Deep arch	Shallow arch
15th cent.	Alberti	$t = s/10$	–
1714	Gautier ($s > 10$ m)	$t = 0.32 + s/15$	–
1777	Perronet	$t = 0.325 + 0.035s$	$t = 0.325 + 0.0694\rho$
1809	Gauthey ($s < 16$ m)	$t = 0.33 + s/48$	–
1809	Gauthey (16 m $\leq s <$ 32 m)	$t = s/24$	–
1809	Gauthey ($s > 32$ m)	$t = 0.67 + s/48$	–
1809	Sganzin	$t = 0.325 + 0.3472s$	–
1845	Déjardin	$t = 0.30 + 0.045s$	$t = 0.30 + 0.025s$
1854	L'Éveillé	$t = 0.333 + 0.033s$	$t = 0.33 + 0.033\sqrt{s}$
1862	Rankine	$t = 0.19\sqrt{R}$	–
1870	Dupuit	$t = 0.20\sqrt{s}$	$t = 0.15\sqrt{s}$
1885	Croizette-Desnoyers	$t = 0.15 + 0.20\sqrt{\rho}$	–
1855	Lesguillier	$t = 0.10 + 0.20\sqrt{s}$	$t = 0.10 + 0.20\sqrt{s}$
1914	Séjourné	$t = 0.15 + 0.15\sqrt{s}$	–

s: span; R: radius of the circle passing through the crown and intrados springing; ρ: curvature radius.

- geometrical survey of Portuguese and Spanish masonry arch bridges supported in the existing literature, with a focus on the adjacent geographical areas of northern Portugal and north-western Spain;
- analysis of the main historical empirical rules used to build bridges and their comparison with survey data;
- definition of reference bridges geometrically representative of the sample;
- numerical assessment of the ultimate load-carrying capacity of the reference bridges, including parametric analysis of the most influential geometrical, physical and mechanical parameters.

The first part of the paper deals with the presentation and discussion of results from the geometrical survey, their comparison with historical empirical rules and definition of the referenced single and multi-span bridges. The second part of the paper focuses on the parametric numerical analysis and discussion of the most important parameters that control the ultimate load capacity of single and multi-span masonry arch bridges.

2. Empirical rules

Prior to the application of statics to masonry arches, initiated by La Hire in the first half of the 18th century, the design process of arch bridges involved the use of empirical rules, which were based on simple geometrical relations and aimed at providing both the dimensions of several bridge components (span, rise and thickness of arch, width and height of piers, etc.) and the safety of the structure based on past experience. Although empirical rules are hardly justified from a mechanical point of view, most of them are revealed to be efficient. Following the works of La Hire, Couplet, Bélidor and many other authors, methods of analysing masonry arch bridges were essentially based on graphic statics. However, establishing the structural form of bridges via an empirical approach continued to be popular, due to builders lacking knowledge of the mathematics and mechanics required to understand and perform static analysis.

2.1. Shape of the arch

The shape of an arch is described as a function of the span s and rise r or, more normally, of the rise to span ratio r/s. Roman arch bridges were typically semicircular ($r/s = 1/2$), though segmental arches ($r/s < 1/2$) were also found. During the mediaeval period, the pointed arch form was introduced. Also, the semicircular shape of Roman arches was reintroduced at the beginning of the Renaissance, but its restricted functionality in urban areas gave rise to new arch forms, which were shallower than the Roman arch. At this time the three-centred arch (basket arch), the ellipse and the inverted catenary were introduced, serving both aesthetic and practical requirements. The S. Trinità Bridge (basket arch) built in 1569 in Italy is the first example of the use of these new forms [20].

2.2. Thickness of the arch

The thickness of an arch bridge can be constant or variable. Typically, arch thickness at the crown was taken from similar existing bridges or based on empirical rules of which there are several, involving varying degrees of complexity. In these, thickness at the crown t is related to span s (or span-related parameters) through different mathematical relationships, but for a detailed discussion and possible classification the reader is referred to Albenga [21] or Proske and van Gelder [2]. Many empirical equations were proposed, mainly during the 19th century. The most well-known expressions [2,20–23] are listed in Table 1 for deep arches only. These equations represent an asymptotic decrease of thickness with thickness to span ratio, not factoring in Alberti's rule. There is reasonable agreement among empirical rules, except for the proposals of Alberti and Gautier. The upper limit is defined by the equation of Lesguillier (19th century) for spans up to 6 m and by the rule of Alberti (15th century) for larger spans, while the lower limit is controlled by the equation of Rankine (19th century).

2.3. Width of piers

The definition of pier width does not depend only on stability issues, being often conditioned by aesthetic aspects. For instance, the minimum geometrical value of pier width for semicircular arches is given by the sum of the thickness of adjacent arches at springing. Furthermore, it is possible that hydrodynamic effects had also been empirically considered [2] in the establishment of the width of piers. In an example of an empirical rule, Campanela [22] states that the width of piers typically varies between 1/5 and 1/10 of the span.

3. Geometrical study

3.1. Geometrical survey

A survey of the most important geometrical properties of ancient roadway masonry arch bridges was carried out based on available literature reviews and previous studies [12–16,24,25]. In total, 59 bridges from Portugal and Spain were considered, with an emphasis on bridges located in the adjacent geographical areas of Minho and Trás-os-Montes provinces (northern Portugal) and the Galicia region (north-western Spain). Indeed, 70% of the surveyed bridges are located in these areas. The predominance of