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Robust sampled-data control of structures subject to parameter uncertainties and actuator saturation

Haiping Du [a,*], Nong Zhang [b,e], Bijan Samali [c], Fazel Naghdy [d]

[a] School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Wollongong, NSW 2522, Australia
[b] Mechatronics and Intelligent Systems, Faculty of Engineering, University of Technology, Sydney, Australia
[c] Infrastructure and the Environment, Faculty of Engineering, University of Technology, Sydney, Australia
[d] School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Australia
[e] State Key Laboratory for Advanced Vehicle Body Design and Manufacturing, Hunan University, China

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ABSTRACT

This paper presents a robust sampled-data controller design approach for vibration attenuation of civil structures considering parameter uncertainties and actuator saturation. The parameter uncertainties belong to polytopic form and are assumed to be the variations of the structural stiffness and damping. Regarding the uncertain sampling problem encountered in real world applications, the sampling period designed for the controller is allowed to be variable within a given bound. In order to obtain reduced peak response quantities, the energy-to-peak performance used to describe the peak values of the control output under all possible energy-bounded disturbances is optimised. The robust sampled-data state feedback controller is obtained in terms of the solvability of certain linear matrix inequalities (LMIs). The applicability of the proposed approach is demonstrated by a numerical example on vibration control of a building structure subject to seismic excitation. It is validated by the simulation results confirming that the designed controllers can effectively attenuate the structural vibration and keep the system stability while there are parameter uncertainties and actuator saturation constraints.

1. Introduction

With the rapid development of computer technology, digital controllers are becoming a reality in many engineering applications, in which a digital computer is used to sample and quantify a continuous-time measurement signal and produce a discrete-time control input signal which is further converted back into a continuous-time control input signal using a zero-order hold. Since physical plants are continuous-time systems in real world, the control systems that use digital controllers involve both continuous-time and discrete-time signals in the continuous-time frame and are referred to as sampled-data systems.

Analysis and synthesis of sampled-data systems have been investigated in a number of papers (see for example [1–5]). In civil engineering, the control of building structures subject to earthquakes or strong winds has received considerable attention over the past three decades and much effort has been devoted to the development of control devices and algorithms [6–8]. With recent focus on wireless monitoring and control of structural systems [9–12] based on networked control technique [13], studying sampled-data control problem for structures is becoming significant. Classical solutions to this type of feedback control problem as well as their applications in civil engineering can be found in the literature [14–16], where optimal discrete-time and sampled-data control algorithms taking into account external excitations were developed for structural engineering applications. Some practical issues such as the effect of sampling frequency, time delay and actuator dynamics were addressed. The methods were numerically validated on the building examples. However, although the effect of sampling frequency was studied in those research and it was shown that the control efficiency were improved significantly with higher sampling frequency, it is noted that the controller design given in those studies is fully dependent on a given sampling rate. That means the controller design is fully based on the assumption that the sampling is made periodic and the controller should be re-designed once the sampling frequency is changed anyway. In practice, the sampling frequency can be varied in terms of the digital realisation requirement and the uncertain sampling may happen when the sampler contains uncertainties or the mathematical model used is not ideally consistent with the sampling equipment. Therefore, designing a sampled-data controller that is robust to the variable sampling rate is necessary.

The parameter uncertainties are one of the most critical issues to a control system as they can affect both the performance and the stability of the control system. Parameter uncertainties may come from modelling errors, variations in material properties, and changing load environments which make the system description for the struc-

* Corresponding author.
E-mail address: hdu@uow.edu.au (H. Du).

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tural models inevitably containing uncertainties of different nature and level [6,17,18]. On the other hand, any actuation mechanisms are subject to inherent physical limitations. The saturation on actuator capacity takes on added importance in structural applications, and in earthquake design in particular [19]. For structures, robust continuous-time controller design considering practical issues like parameter uncertainties, actuator saturation, actuator failure, time delay, etc., was recently studied by, for example, [20–24]. As indicated in the concluding remarks of [16], considering actuator saturation and model uncertainties in the sampled-data controller design process should be a logical next step. This motivates the present study.

This paper concerns with the robust sampled-data controller design for buildings with parameter uncertainties and actuator saturation constraint. The objective is to design a state feedback controller such that the closed-loop system is asymptotically stable with an optimal disturbance attenuation subject to parameter uncertainties and actuator saturation. The parameter uncertainties dealt with are of a polytopic type, the sampling rate is designed to be variable, and the energy-to-peak performance [25] is used to obtain good peak response quantities. Based on the recently developed input delay approach [4,5], sufficient conditions for designing such a controller are derived in terms of linear matrix inequalities (LMIs) which can be resolved efficiently using the available software Matlab LMI Toolbox. To validate the effectiveness of the approach, the designed controllers are applied to reduce the vibration of a seismic-excited building structure. Simulation results show good vibration attenuation performance and system robust stability in spite of parameter uncertainties, actuator saturation, and variable sampling rate.

The rest of this paper is organised as follows. Section 2 presents the problem description for sampled-data control of uncertain structures. Section 3 derives the conditions for designing the robust controller. Section 4 provides an application example to validate the effectiveness of the approach developed. Finally, we conclude our findings in Section 5.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all $n \times m$ real matrices. For a real symmetric matrix W, the notation of $W \succ 0$ ($W \prec 0$) is used to denote its positive-(negative-) definiteness. $\mathbf{I}$ is used to denote the identity matrix of appropriate dimension. When a matrix is equal to $\mathbf{0}$, in such case, $\mathbf{0}$ is used to denote the zero matrix of appropriate dimension. To simplify notation, $*$ is used to represent a block matrix which is readily inferred by symmetry.

2. Sampled-data control of uncertain structure

Consider an n degree-of-freedom (DOF) actively controlled building structure subject to external excitations, the governing equation is written as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Ew(t) + Hu(t), \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$, and $x_n(t)$ is the nth floor relative displacement with respect to ground; $\dot{x}(t)$ and $\ddot{x}(t)$ are the first and second time derivatives of $x(t)$, respectively; $u(t) = [u_1(t), u_2(t), \ldots, u_r(t)]^T$, $u_r(t)$ is the rth control force; $H \in \mathbb{R}^{n\times r}$ gives the location of the r controllers; $w(t)$ is the external excitation; E is a vector denoting the influence of external excitation; $M, C, K \in \mathbb{R}^{n\times n}$ are the mass, damping, and stiffness matrices of the structure, respectively.

Define the state vector as $q(t) = \left[x^T(t)\ \dot{x}^T(t)\right]^T$, the state space representation of the structure in (1) can be expressed as

$$\dot{q}(t) = Aq(t) + B_w w(t) + Bu(t), \tag{2}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}.$$

Consider system (2) has parameter uncertainties, and in particular, the parameter uncertainties are induced by the variations of stiffnesses and damping coefficients, the parameter uncertainties in matrices of system (2) can belong to a polytopic set described by κ vertices, then, the system matrix A can be expressed as

$$A(\xi) \in \Theta \triangleq \left\{ A(\xi) | A(\xi) = \sum_{i=1}^{\kappa} \xi_i A_i; \xi_i \geqslant 0; \sum_{i=1}^{\kappa} \xi_i = 1 \right\} \tag{3}$$

where ξ is used to characterise the parameter uncertainty and is assumed to be varied in a polytope of vertices $\xi_1, \xi_2, \ldots, \xi_\kappa$, i.e., $\xi \in \Theta \triangleq Co\{\xi_1, \xi_2, \ldots, \xi_\kappa\}$, where the symbol Co denotes the convex hull and Θ denotes a given convex bounded polyhedral domain.

With further consideration on the actuator saturation, system (2) is expressed as

$$\dot{q}(t) = A(\xi)q(t) + B_w w(t) + B \cdot \text{sat}(u(t)), \tag{4}$$

where the actuator saturation expression sat(u) is in the decentralised saturation form, that is, $[\text{sat}(u)]_i = \text{sat}(u_i)$, where $i = 1, 2, \ldots, r$, and sat(u_i) is the standard saturation function with the limit of $u_{\lim_i}$ for the ith actuator, that is,

$$\text{sat}(u_i) = \begin{cases} u_i, & |u_i| \leqslant u_{\lim_i}, \\ \text{sign}(u_i)u_{\lim_i}, & |u_i| > u_{\lim_i}. \end{cases} \tag{5}$$

Using the following transform [26–28]

$$\text{sat}(u) = \Psi(\eta)u, \tag{6}$$

where $\Psi(\eta) = \text{diag}\{\eta_1, \ldots, \eta_i, \ldots, \eta_r\}$, $\eta_i \triangleq \frac{\text{sat}(u_i)}{u_i}$ with $\eta_i = 1$ if $u_i = 0$, Eq. (4) can now be written as

$$\dot{q}(t) = A(\xi)q(t) + B_w w(t) + B\Psi(\eta)u(t). \tag{7}$$

To obtain a high gain controller as that in [26], the command to the ith actuator is allowed to be $\delta_i u_{\lim_i}$ for an arbitrary scalar $\delta_i > 1$. Therefore, the resulting η_i will be bounded by 1 and $1/\delta_i$, that is,

$$\eta \in \mathcal{P} \triangleq \left\{ \eta : \frac{1}{\delta_i} \leqslant \eta_i \leqslant 1,\ i = 1, 2, \ldots, r \right\}. \tag{8}$$

Accordingly, the vertex set associated with (8) is denoted as

$$\mathcal{P}_{vex} \triangleq \left\{ \eta : \eta_i = \frac{1}{\delta_i} \quad \text{or} \quad \eta_i = 1,\ i = 1, 2, \ldots, r \right\}, \tag{9}$$

and $\Psi(\eta)$ can be expressed as $\Psi(\eta) = \sum_{i=1}^{2r} \zeta_i \Psi(\eta_i) = \sum_{i=1}^{2r} \zeta_i \Psi_i$, where $\zeta_i \geqslant 0$ and $\sum_{i=1}^{2r} \zeta_i = 1$.

In this paper, the external excitation signal $w(t)$ is assumed to be bounded and with finite energy, that is,

$$\|w\|_2 \triangleq \sqrt{\int_0^\infty w^T(t)w(t)dt} < \infty, \tag{10}$$

i.e., $w(t) \in L_2[0, \infty)$. This is one possible specification for a class of design loads that the engineering structures are designed to resist, for example, a class of design earthquakes whose intensity and associated total energy is specified on a Richter scale [25].

To design a controller for active vibration attenuation of structures under external excitations, the control output should be defined so that the performance index from the external excitation to the control output can be realised with the specified requirement. For system (7), we define the control output as

$$z(t) = C_z q(t), \tag{11}$$

where C_z is a constant matrix which defines the interested output variables.

Now, it is assumed that the state variables of the building structure are measured at time instants $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$, and only $q(t_k)$ are available for interval $t_k \leqslant t < t_{k+1}$. Then, for the