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Buckling of flat laminated glass panels under in-plane compression or shear

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ABSTRACT

Because of evident aesthetic, lighting and architectural advantages, glass curtain walls are largely used to clad modern buildings. Since these elements are considered to constitute purely architectural systems, they are essentially designed to resist loads acting orthogonally to the plane of the façade (e.g. wind loads). Contrarily, glass elements are frequently used as structural components able to sustain in-plane loads (e.g. columns, stiffening fins, beam elements, stairs, etc.), thus to preserve their integrity a buckling verification could assume great importance.

In order to overcome these problems, an analytical formulation is proposed for the estimation of the buckling resistance of flat laminated glass panels under in-plane compression or shear. Two different design approaches are taken into account and compared: the first one directly derives from the theory of sandwich panels, whereas the second one is based on the approximate concept of equivalent thickness. As discussed in the paper, this last approach constitutes a useful design expedient for the deformability and resistance check of buckled laminated panels under in-plane compression or shear, in presence of different boundary conditions. Since the resistance of such brittle elements directly depends on the level of connection between the glass panes offered by the interlayer, the effects of possible temperature and time-loading variations are highlighted. The obtained analytical results are in agreement with sophisticated numerical simulations.

1. Introduction

In the last years, the use of laminated glass (LG) elements in the realization of façades, stiffening fins, roofs, stairs, columns, etc., showed a strong increase. In this manner, a construction typology originally used as a purely architectural and decorative accessory in buildings, is rapidly becoming a fundamental structural component in the realization of futuristic and innovative architectures. Although the main motivations of such diffuse structural use of glass in modern buildings still derive from aesthetic and architectural motivations, additional advantages are implicitly provided by this type of elements, specially by laminated glass beams or panels (sound insulation, lightening properties, post-breakage behavior, etc.).

Generally, due to their typical slenderness, LG panels subjected to in-plane loads are affected by stability problems, consequently an appropriate verification criteria should prevent failure mechanisms associated to this possible limit condition. In the specific case of glass–steel façades components, it is possible to distinguish specific structural behaviors respectively associated to stability problems due to in-plane loads (e.g. pure compression, shear, or a combination of them), eventually associated to out-of-plane loads (e.g. wind, seismic event,...).

Several authors recently investigated the buckled response of glass panels or beams subjected to in-plane compression [1], out-of-plane bending [2], concentrated in-plane forces [3], in-plane shear [4], providing useful experimental data, sophisticated numerical validations and interesting analytical considerations. Nevertheless, the knowledge on LG panels behavior under in-plane loads is still limited and with constrained applications.

For the buckling verification of traditional structural elements, realized by means of conventional materials as steel or concrete, consolidate verification criteria are available in literature. Contrarily, existing methods cannot be directly applied to LG elements, because they do not take into account a series of well-known factors (influence of production tolerances, initial imperfections, brittle behavior of glass and viscoelastic behavior of thermoplastic interlayers typically used). For the buckling verification of LG beams under compression or out-of-plane bending, an analytical approach which requires the contemporary check of maximum stresses, deformations and acting loads has recently been proposed [5,6]. In the case of LG panels, contrarily, to perform a realistic buckling verification and to describe their post-critical behavior is frequently necessary to adopt sophisticated numerical models.

In the paper an analytical approach is proposed for the buckling verification of LG panels under in-plane compression or shear. The

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modeling procedure is based on the concept of equivalent thickness originally formulated by Wölfel [7] and allows to evaluate, by means of a series of appropriate numerically calibrated coefficients, the critical buckling load of simply supported laminated panels under in-plane compression or shear. At the same time, the proposed analytical approach allows to describe the corresponding load–transversal displacement by taking into account the effective level of connection between the glass sheets offered by the interlayer. Specifically, the model describes the elastic buckling and the post-critical behavior of LG panels in presence of well-defined conditions of temperature and load duration by taking into account specific mechanical properties for the interlayer, thus the use of viscoelastic theories and FEM simulations can be avoided.

Existing formulations concerning the buckling behavior of sandwich panels under in-plane compression or shear [8,9], because of their hypotheses, frequently cannot be directly applied to laminated glass elements. In contrary, the proposed methodology provides accurate results also for laminated panels characterized by very soft interlayers, as demonstrated by the series of analytical and numerical comparisons presented in the paper.

Analytical formulations are proposed in order to individuate the notorious limit behaviors of laminated panels (*layered limit*, *monolithic limit*) and to estimate how possible temperature or load-duration modifications can reduce the effective level of connection. By means of a numerical calibration of opportune buckling coefficients, the proposed approach is extended to in-plane compressed LG panels subjected to different boundary conditions. Also in this circumstance, the comparison between analytical and numerical results allows to highlight the simplicity and the accuracy of the proposed method.

At last, a rational procedure for the buckling verification of laminated glass panels is proposed, requiring the contemporary satisfaction of two different limit conditions, concerning respectively the check of deformability and resistance.

2. Simply supported laminated glass panels under compression

2.1. Analytical models for monolithic and laminated panels

The critical buckling load $N_{y,cr}$ (force per unit length acting in y-direction) of a perfectly flat monolithic glass panel (thickness t, Young's modulus E) subjected to purely in-plane uniform compression N_y, simply supported along its four edges (Fig. 1), is commonly evaluated by means of classical analytical models, based on the linear elastic bending theory concerning the behavior of flat homogeneous and isotropic elements.

Referring to Fig. 1 and assuming that the following hypotheses are satisfied:

- in the undeformed configuration the panel is perfectly flat;
- the displacements in x and y-directions are negligible along the edges of the panel;
- the shear deformations in the panel can be ignored;
- $N_x = N_{yx} = N_{xy} = 0$;

the out-of-plane displacement $w = w(x, y)$ of the panel due to an external pressure force per unit length $N_y = \sigma_y t$ can notoriously be described by means of the differential equation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \Delta^2 w = \frac{N_y}{D}\frac{\partial^2 w}{\partial x^2}, \tag{1}$$

where, Δ, is the Laplace's operator; $D = Et^3/12(1-\nu^2)$, is the flexural stiffness of the element, per unit width b; ν, is the Poisson's coefficient for glass.

From Eq. (1), the critical buckling load results expressed by the well-known expression [10]:

$$N_{y,cr} = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \frac{\pi^2 D}{b^2} = k_\sigma \frac{\pi^2 D}{b^2}, \tag{2}$$

where, a and b, are respectively the length and the width of the panel (Fig. 1); m, represents the number of half waves in y-direction; k_σ, is the buckling coefficient.

Similar considerations can be extended to the study of buckled LG panels under in-plane compression, although in the specific circumstance, their buckling resistance is generally estimated referring to the common linear elastic theory of sandwich elements. In [1], for example, Luible proposes Zenkert's formulation [11], which is based on the following assumptions:

- both the materials of the middle core and the external faces of the composite panel are elastic;
- the external faces are flat and have a constant thickness;
- the flexural stiffness of the external faces cannot be neglected;

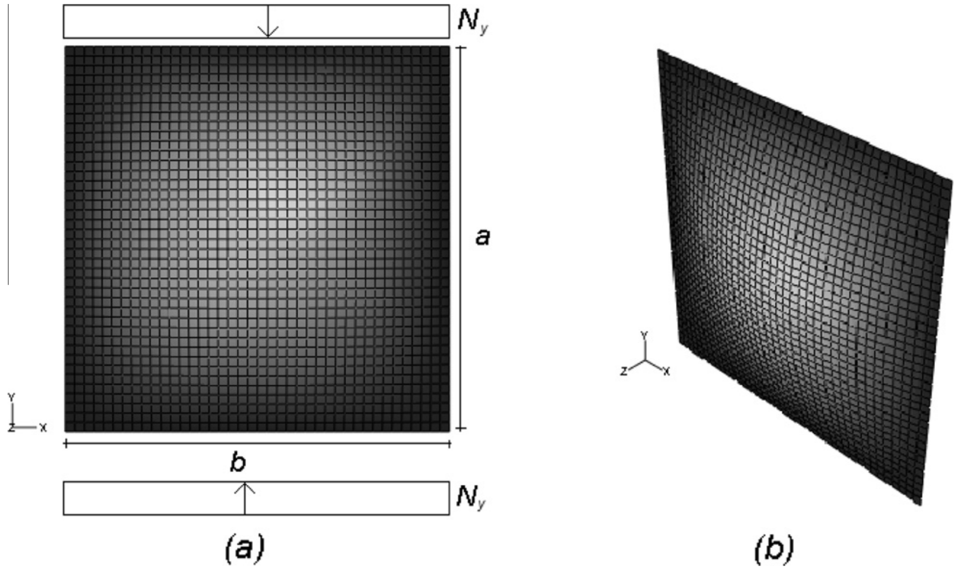

Fig. 1. Simply supported flat panel subjected to in-plane compression. (a) Geometry; (b) deformed configuration.