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Consistent virtual work approach for the nonlinear and postbuckling analysis of steel frames under thermal and mechanical loadings

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ABSTRACT

The aim of this paper is to provide a consistent virtual work formulation for the nonlinear and postbuckling analysis of steel frames at high temperatures. Central to this study is the derivation of the virtual work terms for the *thermal stage*, in addition to those for the *loading stage*, based on the updated Lagrangian formulation. The incremental stiffness equation derived for the beam element, considering both the geometrical and thermal effects, is qualified by the rigid body test. The generalized displacement control (GDC) method is adopted as the path-tracing scheme for postbuckling response. Eurocode-3 reduction factors and transformed section method are both adopted for steel I-sections. Two loading cases are studied. For structures loaded gradually under constant temperature, the critical or ultimate loading strength is obtained from the load-deflection curve. For structures heated gradually under constant loading, the critical or maximum temperature that can be sustained by the structure is computed. Conclusions are drawn for the examples studied in this paper.

1. Introduction

Fire disaster is becoming a noticeable problem in populated metropolitan areas, which is coupled by the increasing use of steel structures. Compared with other structural materials, such as concrete, steel has a high thermal conductivity, which softens rapidly when the temperature reaches some values. Once the critical state is reached, the structure may not collapse immediately, but the overall structural safety may be seriously affected. In this regard, how to simulate the behavior of steel structures under a major fire is crucial to assessment of the remaining strength of a damaged structure for rehabilitation for further use.

Both fire resistance tests and numerical simulations methods, particularly the finite element method, have been employed in evaluating the fire resistance capability of steel structures and components. The fire tests are generally costly and subject to certain physical restraints, such as the furnace environment, member constraints, and so on. In contrast, the finite element method is generally versatile, by which various factors such as non-uniform temperature distribution, geometrical and material nonlinearities, etc., can be easily taken into account [1–6]. The results obtained by a finite element program are often compared with those from the fire tests. But this has been quite limited due to the restraint for preparing the specimens for use inside the furnace and other physical restraints. To ensure the general applicability of a finite element procedure, it is necessary to develop some benchmark problems for which the solutions can be used as the baselines [7].

There exists an abundant literature on the finite element simulation of the behavior of steel frames in fire. Li and Jiang [8] investigated the behavior of steel frames by considering the material and geometrical nonlinearities, and the temperature distribution across member sections. By using the generalized Clough model, the tangent stiffness at high temperature can be obtained and the effect of thermal strain is converted to equivalent thermal loads at structural nodes. However, the geometric and thermal stiffness matrixes were not qualified by the rigid body test described in [9,10]. Iu et al. [11] used the energy method to obtain the incremental force–displacement relationship, and the Newton–Raphson method to study the nonlinear behavior of steel frames with no protection cover. In their analysis, effects such as large deformations, plastic hinges, and strain hardening are included. Yin and Wang [5] used the ABAQUS program to analyze the large deformation behavior of steel frames with different constraints under the heating stage. The parameters considered include the span length, uniform and non-uniform temperature distributions, different loading conditions, rotation constraint, and lateral buckling.

In this paper, a consistent virtual work theory is presented for the nonlinear and postbuckling analysis of steel frames at high temperatures. Central to this study is the derivation of

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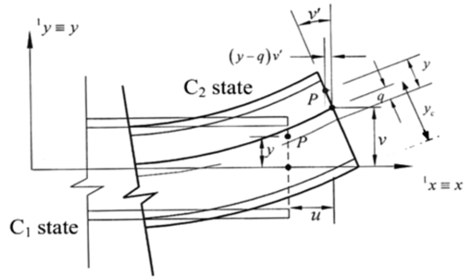

Fig. 1. Plane section assumption for two-dimensional beam.

the virtual work terms for the *thermal stage*, in addition to those for the *loading stage*, based on the updated Lagrangian formulation. The incremental stiffness equation derived for the steel beam, considering both the geometrical and thermal effects, is qualified by the rigid body test. The transformed section method is adopted to account for non-uniform temperature distribution of the steel I-section. The generalized displacement control (GDC) method [12] is adopted as the incremental-iterative scheme for tracing the postbuckling behavior of structures. For structures under *constant temperature*, the critical load is solved from the load–deflection curve. For structures under *constant loading*, the critical temperature is obtained from the temperature–deflection curve.

2. Kinematics and statics of the beam

With the updated Lagrangian (UL) formulation, the last configuration C_1 is selected as the reference configuration for setting up the equation of equilibrium for the beam element at the current configuration C_2. Based on the Bernoulli–Euler hypothesis of plane sections remaining plane and normal to the centroidal axis of the beam after deformation, the displacements increments u_x and u_y of a generic point N at section x of the beam during the incremental step from C_1 to C_2 are (Fig. 1):

$$u_x = u - (y-q)v', \tag{1a}$$
$$u_y = v \tag{1b}$$

where u and v denote the displacements of the centroid at section x, v' the rotational angle at section x, and q the distance due to shift of the centroid from C_1 to C_2 upon thermal rise,

$$q = y_c - \frac{h}{2} \tag{2}$$

in which y_c denotes the distance of the centroid at C_2 to the bottom side of the section, and h is the section's depth.

2.1. Strain and stress increments from C_1 to C_2

2.1.1. Strain increments

The updated Green strain increment ${}_1\varepsilon_{xx}$ can be decomposed into the linear and nonlinear components as follows:

$${}_1\varepsilon_{xx} = {}_1e_{xx} + {}_1\eta_{xx} \tag{3}$$

in which ${}_1e_{xx}$ denotes the linear component of the strain increment,

$${}_1e_{xx} = u_{x,x} \tag{4}$$

where a comma denotes differentiation with respect to the following coordinate. By the use of Eq. (1a), the linear strain component ${}_1e_{xx}$ can be written as

$${}_1e_{xx} = u' - (y-q)v''. \tag{5}$$

The nonlinear strain component of the strain ${}_1\eta_{xx}$ is

$${}_1\eta_{xx} = \frac{1}{2}(u_{xx}^2 + u_{yx}^2). \tag{6}$$

By using Eq. (1b) and neglecting the first term in Eq. (6), the nonlinear strain component ${}_1\eta_{xx}$ can be written as

$${}_1\eta_{xx} = \frac{1}{2}v'^2. \tag{7}$$

Thus, the total strain increment ${}_1\varepsilon_{xx}$ is

$${}_1\varepsilon_{xx} = u' - (y-q)v'' + \frac{1}{2}v'^2. \tag{8}$$

Similarly, the shear strain increment ${}_1\varepsilon_{xy}$ can be decomposed into the linear and nonlinear components as

$${}_1\varepsilon_{xy} = {}_1e_{xy} + {}_1\eta_{xy}. \tag{9}$$

With the substitution of the displacements in Eq. (1), the linear strain component ${}_1e_{xy}$ can be shown to vanish,

$${}_1e_{xy} = \frac{1}{2}(u_{x,y} + u_{y,x}) = 0. \tag{10}$$

Meanwhile, the nonlinear shear strain component ${}_1\eta_{xy}$ is

$${}_1\eta_{xy} = \frac{1}{2}(u_{x,y}u_{x,x} + u_{y,y}u_{y,x}) = \frac{1}{2}[-u'v' + (y-q)v'v'']. \tag{11}$$

By combining Eqs. (10) and (11), the shear strain increment ${}_1\varepsilon_{xy}$ can be obtained as

$${}_1\varepsilon_{xy} = \frac{1}{2}[-u'v' + (y-q)v'v'']. \tag{12}$$

2.1.2. Stress increments

The stress increment ${}_1S_{xx}$ at a generic point of section x can be expressed as

$${}_1S_{xx} = {}^1_1E({}_1\varepsilon_{xx} - \varepsilon_T) \tag{13}$$

in which ${}^1_1E$ denotes the elastic modulus at C_1 and ε_T the strain increment induced by the temperature increment,

$$\varepsilon_T = \alpha\left[{}^2T_t + (y-q)\frac{\Delta T_a}{h}\right] \tag{14}$$

in which ΔT_a represents the temperature difference between the top and bottom flanges at C_2,

$$\Delta T_a = {}^2T_b - {}^2T_t. \tag{15}$$

According to Eq. (10), the shear stress increment ${}_1S_{xy}$ simply vanishes,

$${}_1S_{xy} = 0. \tag{16}$$

2.2. Initial forces acting at C_1

For a two-dimensional bean, the initial forces of each cross-section, i.e., the axial force ${}^1F_x$, transverse shear ${}^1F_y$, and bending moment ${}^1M_z$, can be related to the initial stresses existing on the beam as

$${}^1F_x = \int_{{}^1A} {}^1\tau_{xx}\mathrm{d}A, \tag{17a}$$
$${}^1F_y = \int_{{}^1A} {}^1\tau_{xy}\mathrm{d}A, \tag{17b}$$
$${}^1M_z = \int_{{}^1A} y\,{}^1\tau_{xx}\mathrm{d}A \tag{17c}$$

where ${}^1\tau_{xx}$ and ${}^1\tau_{xy}$ denote that axial and shear (Cauchy) stresses, respectively, and ${}^1A$ denotes the cross-sectional area of the beam.