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A neural network based closed-form solution for the distortional buckling of elliptical tubes

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ABSTRACT

Following the Eurocode 3 philosophy, it is expected that the design of elliptical hollow section (EHS) tubes will be based on the slenderness concept, which requires the calculation of the EHS critical stress. The critical stress of an EHS tube under compression may be associated with local buckling, distortional buckling or flexural buckling. The complexity in deriving analytical expressions for distortional critical stress from classical shell theories, led us to apply Artificial Neural Networks (ANN). This paper presents closed-form expressions to calculate the distortional critical stress and half-wave length of EHS tubes under compression, using ANN. Almost 400 EHS geometries are used and based solely on three parameters: the outer EHS dimensions (A and B) and its thickness (t). Two architectures are shown to be successful. They are tested for several statistical parameters and proven to be very well behaved. Finally, some simple illustrative examples are shown and final remarks are drawn concerning the accuracy of the closed-formed formulas.

1. Introduction

1.1. Literature review

Since the beginning of the second half of the 20th century, the evaluation of critical buckling stress of non-circular cylindrical shells has been a major concern to structural engineers. The first investigation into the buckling behavior of non-circular shells under uniform compression is credited to Marguerre [1] while consistent investigations on the buckling and post-buckling behavior of oval shells were performed by Kempner and his collaborators [2–5]. In their studies, they concluded that (i) the buckling mode maximum deflection occurs at the point of maximum radius of curvature and (ii) the buckling stress of an oval shell is similar to the buckling stress of an equivalent circular shell with a radius equal to the maximum radius of curvature of the oval shell. Moreover, Feinstein et al. [4,5] observed that the buckling stress of oval shells depended not only on their aspect ratio a/b ($2a$ and $2b$ are the major and minor axis widths with respect to the cross-section mid-line—see Fig. 1) but also on the length of the shell. The first investigation on the buckling behavior of elliptical shells appeared in 1968 due to the intense work of Hutchinson [6], who concluded that elliptical shells are imperfection-sensitive (like circular shells) but the buckling phenomenon is not catastrophic (as in circular shells) and failure may even occur at a higher stress than the critical buckling stress. Tennyson et al. [7] also found experimentally that elliptical shells with moderate to high eccentricity ($a/b \geq 2$) exhibited ultimate stresses higher than their critical stresses. This was attributed to the fact that highly eccentric elliptical shells display a post-buckling behavior more close to the plate behavior (stable) than circular shell behavior (unstable). After the pioneering works of Kempner and Hutchinson, not much has been done in the 80s and 90s.

In the beginning of the 21st century, steel tubes with elliptical hollow section (EHS) become available in the market [8] and raised the interest of the technical and scientific communities. Nowadays, steel EHS represent a quite interesting solution for all visible applications in steel construction, particularly for glass roofs and glass façades. They have been used in major construction steelworks, such as the Barajas airport in Madrid, Heathrow airport in London and the Coeur Défense atrium in Paris. Since 2004, several investigations on the EHS tube buckling behavior and strength have been published and well resumed by Packer [9]. At Imperial College London, Gardner and colleagues devoted extensive research to the behavior and design of EHS tubes. Chan and Gardner [10,11] developed design guidance for carbon steel EHS tubes, Theofanous et al. [12] investigated the

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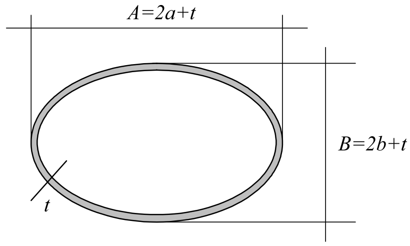

Fig. 1. Elliptical hollow section (EHS) geometry.

behavior of stainless steel EHS tubes in compression and Ruiz-Teran and Gardner [13] examined the elastic buckling response of EHS tubes and proposed analytical formulas to accurately predict the local critical stress. More recently, Silvestre and Gardner [14] investigated the post-buckling and strength of EHS stub columns. Additionally, the behavior and strength of concrete-filled EHS columns has also been investigated. Roufegarinejad and Bradford [15] used an energy based technique to investigate the local buckling of uniformly compressed EHS tubes with an elastic infill. Yang et al. [16] and Zhao and Packer [17] performed experimental tests on concrete-filled EHS columns and proposed design guidance. Finally, the work of Nowzartash and Mohareb [18] on EHS also deserves to be mentioned. They developed lower bound interaction relations for EHS tubes under combined action of axial force, bi-axial bending moments, and twisting moments based on statistically admissible stress fields.

1.2. EC3 design background and buckling behavior of EHS tubes

Before we describe the buckling behavior of EHS tubes under compression, let us show the current rules for the design of CHS tubes, which are a particular case of EHS with $a = b = r$. In accordance with the cross-section classification in Eurocode 3 Part 1.1 [19], a CHS is of class 4 if the following criterion is satisfied,

$$\frac{D}{t\varepsilon^2} = \frac{D}{235 \cdot t} f_y > 90 \tag{1}$$

where $D = 2r$ and t are the diameter and thickness of the CHS and f_y is the steel yield stress. For a class 4 CHS, Eurocode 3 Part 1.1 [19] also states that its ultimate strength should be determined by means of Eurocode 3 Part 1.6 [20], for the design of shell structures. Using the Buckling Limit State (LS3 in Part 1.6 of EC3), the ultimate stress f_u of a class 4 CHS tube is given by

$$f_u = \begin{cases} f_y & \text{if } \bar{\lambda} \le 0.2 \\ f_y \left(1 - 0.6 \dfrac{\bar{\lambda} - 0.2}{\bar{\lambda}_p - 0.2}\right) & \text{if } 0.2 < \bar{\lambda} < \bar{\lambda}_p \\ \dfrac{\alpha}{\bar{\lambda}^2} f_y = \alpha \sigma_{cr} & \text{if } \bar{\lambda} \ge \bar{\lambda}_p \end{cases} \tag{2}$$

where $\bar{\lambda}_p$ is the plastic limit relative slenderness, which depends on the imperfection reduction factor α (knockdown factor), and $\bar{\lambda}$ is the CHS tube relative slenderness given by

$$\bar{\lambda} = \sqrt{\frac{f_y}{\sigma_{cr}}}. \tag{3}$$

Eq. (3) shows that the elastic critical stress σ_{cr} of the CHS tube is deemed necessary for the evaluation of the CHS tube ultimate.

Regarding EHS tubes, Chan and Gardner [10,11] performed extensive research on their behavior, strength evaluation and design guidance. Using a modification of the EC3 Part 1.1 classification criterion for CHS, they proposed that Class 4 EHS should satisfy the criterion

$$\frac{D_{\max}}{t\varepsilon^2} = \frac{2a^2 f_y}{235 \cdot bt} > 90 \tag{4}$$

where $D_{\max} = 2r_{\max} = 2a^2/b$ is the maximum diameter of the EHS. Conversely to the classification of EHS, design provisions for the EHS tube ultimate strength evaluation still do not exist in current codes. Following the Eurocode 3 philosophy (as in the case of CHS tubes), it is expected that the design of EHS tubes will also be based on the relative slenderness concept, which requires the calculation of EHS tube critical stress σ_{cr}.

It has been shown by Silvestre [21] that the buckling of EHS tubes is governed by three distinct buckling modes: (i) local buckling mode for short lengths, (ii) distortional buckling mode for intermediate lengths and (iii) flexural (Euler) mode for long lengths, the first two modes being characterized by deformation of the tube cross-section contour (see Fig. 2). The distinctive feature is that the half-wave length of the distortional buckling mode is much higher than the half-wave length of the local buckling mode.

The available expressions for the critical stress calculation are based on the modified Donnell formula for CHS tubes, replacing the CHS radius by the maximum radius of the EHS ($r_{\max} = a^2/b$). A lower but often accurate bound of the critical stress for local buckling of EHS tubes (Fig. 1) is given by the formula,

$$\sigma_{cr.L} = \frac{E}{\sqrt{3(1-\nu^2)}} \left(\frac{t}{a^2/b}\right). \tag{5}$$

Silvestre [22] derived a formula to calculate the local half-wave length of CHS tubes. Replacing the CHS radius in that formula by the maximum radius of the EHS ($r_{\max} = a^2/b$), the very short half-wave length of EHS local buckling is obtained

$$L_{cr.L} = \frac{\pi a \sqrt{t/b}}{\sqrt[4]{12(1-\nu^2)}}. \tag{6}$$

Based on Eq. (1), more refined expressions to estimate the value of $\sigma_{cr.L}$ were proposed by Ruiz-Teran and Gardner [13] and Silvestre [21]. The formula given in Eq. (1) works well for local buckling since it is triggered by the instability of the less stiff regions of the EHS tube (see Fig. 2), where the buckling lobes are located and the radius achieves its maximum value ($r_{\max} = a^2/b$). It has been shown that the modified Donnell formula does not give good predictions of the distortional critical stress of EHS tubes [21]. The exact calculation of the distortional critical stress is more complex and simple formulas are not yet available. This evidence could be mainly explained by the complex distortional buckling kinematics, which primarily involves warping along the EHS mid-line, which is absent in local buckling modes.

The main purpose of this paper is to overcome such a difficulty using Artificial Neural Networks (ANNs) to obtain a closed-form solution for the prediction of distortional buckling stress and half-wave length of EHS tubes under compression. With this closed-form expressions, the structural designers (i) have a tool to estimate the distortional critical stress and half-wave length of EHS tubes, (ii) may avoid unnecessary performance of shell finite element or finite strip simulations and (iii) use future design guidance for EHS tubes, which will inevitably be based on the value of σ_{cr} (in accordance with EC3 philosophy).

2. Neural networks

2.1. Brief overview and scope

Artificial Neural Networks (ANNs) have been successfully employed in solving complex problems in several fields of application,