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A meshfree model without shear-locking for free vibration analysis of first-order shear deformable plates

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ABSTRACT

Difficulty in imposing essential boundary conditions in the standard element-free Galerkin method (EFG) is due to the lack of Kronecker's delta function property of shape functions generated by moving least square approximation (MLS). In this paper, we further apply a meshfree model based on the moving Kriging interpolation method (MK) to free vibration analysis of first-order shear deformable plates. The deflection and two rotation field variables of plate are approximated by the MK method, which is employed to construct the shape functions having the delta function property. With this approach, the drawback in enforcement of the boundary conditions caused by the MLS is now avoided. The present formulation is based on the first-order shear deformation plate theory (FSDT) associated with an effective elimination of the shear-locking phenomenon completely, and hence the approach is applicable to both moderately thick and thin plates. Numerical examples considering various aspect ratios and different boundaries are examined and solutions on natural frequencies obtained by the present method are then compared with existing reference solutions, and very good agreements are observed.

1. Introduction

Plate structures have been intensively used in a variety of engineering disciplines involving civil engineering, automobiles, aerospace, construction sectors, marine, naval, etc., but a thorough understanding of their vibration characteristics is of great importance to engineers and designers making sure reliability in design procedure. The great majority of solutions existing for the flexural vibration of plates at the beginning are based on the classical Kirchhoff assumption [1] which neglects the transverse shear deformation of the plates during the process, and the rotary inertia terms are also ignored. The absence of those characteristics firmly leads to the overestimation of the plate frequencies and significant errors are increased when the thickness-span ratio is increased. A substantial development of plate theory, taking into account the effect of such transverse shear deformation and rotary inertia, was proposed by Reissner [2,3] for the first-order shear deformable theory (FSDT). Mindlin [4–6] later presented a variational approach deriving the governing plate equation for free vibration of the FSDT incorporated the rotary inertia effect. Obtaining analytical solutions for free vibration problems of Reissner–Mindlin plates is more difficult due to more governing equations and kinetic parameters involved [7–14]; thus, approximate solutions with a high level of accuracy using numerical computational approaches are indispensable.

Many numerical methods have been introduced and successfully applied to free vibration analysis of Reissner–Mindlin plates such as Rayleigh method [15], Rayleigh–Ritz methods [16–20], *pb-2* Rayleigh–Ritz methods [21,22], spline strip method (SSM) [23], finite strip method (FSM) [24,25], spline finite strip method (SFSM) [26–28], boundary element method (BEM) [29], generalized differential quadrature method (GDQ) [30,31], discrete singular convolution (DSC) method [32–37], DSC-Ritz method [38], finite element method (FEM) [39–42], etc.

Although these numerical methods have been demonstrated accurately and efficiently in solving such plate vibration problems, their disadvantages are always present for each approach and they still have some limitations in engineering applications. As stated in [36], the structural computations are generally accomplished by employing either global or local methods. The global approach such as Rayleigh, Rayleigh–Ritz, GDQ is highly accurate but often cumbersome in treatment of general boundary conditions and complex geometries, and in contrast, the SSM, FSM, SFSM, etc., standing for the local ones, which are easy

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to deal with complex geometries and discontinuous boundary conditions but their accuracy is relatively low compared with that of the global ones. Sometimes, the convergence in terms of the local approach is slow and expensive for short waves, i.e. high-order eigenmodes [43]. Recently, the DSC and DSC-Ritz methods have shown some advantages in accuracy and flexibility compared to those previously presented such as can handle complex geometry and boundary conditions, and vice-versa, the Rayleigh–Ritz method is difficult to choose the appropriate trial functions for complicated problems. The BEM is also very difficult to find an appropriate fundamental solution for complicated problems and in fact, the FEM is really effective, flexible and one of the most popular methods for engineering applications nowadays but there are some problems related to meshing, distortion, discontinuities and so on. In recent years, a promising numerical technique emerging alternatively named meshfree or meshless method, e.g. see [44–48], has been introduced and shown some advantages superior over the traditional numerical methods. The common concepts of elements or meshing used in the FEM are no longer required in meshfree methods, and only nodes scattered in the domain of problems is used for approximating field variables instead.

Meshless methods have been successfully applied to plate problems in recent years. Thin plates based on Kirchhoff's assumption have been analyzed by the element-free Galerkin (EFG) method [44,49], the meshless local integral equation method [50,51] and the meshless local Petrov–Galerkin (MLPG) method [52], just to name a few. In a similar manner, the reproducing kernel particles method (RKPM) [53], the MLPG method [54–57], the EFG method [44,58,59], the meshless radial point interpolation method (RPIM) [60], and many other variants have also been applied to the analysis of Reissner–Mindlin plates with a moderate thickness. An important phenomenon when using thick plate theories to analyze thin plates is shear-locking. The derivation of the shear-locking in the Reissner–Mindlin plate formulation due to either inability in the approximation functions to reproduce the Kirchhoff mode or the incapability of numerical methods to achieve pure bending exactness in the approximation [61]. The techniques eliminating this shear-locking have been well developed in the FEM, e.g., see [62–65], and various approaches have also been proposed in meshfree methods, for instance, higher-order basis functions are employed in *h-p* cloud method [66]; a stabilized conforming nodal integration technique is applied to both the moving least square (MLS) and reproducing kernel (RK) approximations [61]; using approximation functions for rotational degrees of freedom (DOF) as the derivatives of the approximation function for translational DOF has also been introduced to resolve the shear-locking [67,68]. In this paper, the technique proposed by Kanok-Nukulchai and his co-workers [68] is applied to our present formulation to eliminate the shear-locking.

Most meshfree methods have the same problems in imposing essential boundary conditions because of the lack of Kronecker's delta property functions of shape functions. The imposition of prescribed values is thus not as straightforward as done in the FEM. For this reason, many efforts have been devoted and some of special techniques have been proposed to overcome such difficulty in various ways e.g. Lagrange multipliers [45], penalty methods [44,69], or coupling with the FEM [70–73], etc. The present formulation possesses the delta property and is thus capable of avoiding such shortcoming completely. The MK technique associated with the EFG method was first presented by Gu [74] for solving a simple problem of steady-state heat conduction. Further developments of the method can be found, respectively, for two-dimensional plane problems [75,76], shell structures [77], static deflections of thin plates [78], piezoelectric structures [79], dynamic analysis of structures [80] and buckling and bending of orthotropic plates [81].

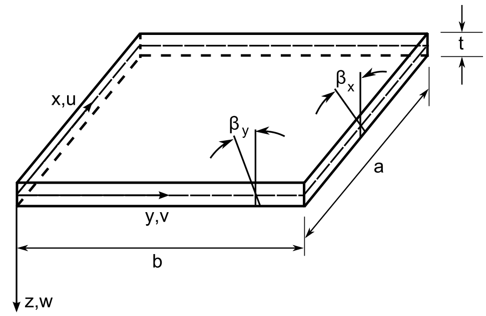

Fig. 1. Geometric notation of a Reissner–Mindlin plate.

The main objective of the paper is to further apply the meshfree moving Kriging interpolation method to the eigenvalue analysis of Reissner–Mindlin plates, which has the delta function property and free of shear-locking. In the following, meshless formulation for free vibration analysis of Reissner–Mindlin plates is presented in the next section presenting a brief description of governing equations and their weak form, and approximation of field variables as well as the discrete equation systems. Numerical results on natural frequencies obtained by the present approach for square and circular plates are investigated and discussed in detail.

2. Meshless formulation for free vibration of plates

2.1. Governing equations and weak form

Consider a Reissner–Mindlin plate as depicted in Fig. 1 with the plate thickness t, two-dimensional mid-surface $\Omega \subset \Re^2$, boundary $\Gamma = \partial\Omega$ and transverse coordinate z. The Reissner–Mindlin theory does not demand the cross section to be perpendicular to the neutral plane after deformation. Consequently, the displacements u and v parallel to the undeformed neutral surface at a distance z from the neutral plane can be defined by [53]

$$\begin{aligned} u &= z\beta_x(\mathbf{x}) \\ v &= z\beta_y(\mathbf{x}) \end{aligned} \tag{1}$$

with $\mathbf{x} = [x, y]^{\mathrm{T}}$ and independent angles $\boldsymbol{\beta}^{\mathrm{T}} = [\beta_x, \beta_y] \in (H_0^1(\Omega))^2$, where $\beta_x(\mathbf{x})$ and $\beta_y(\mathbf{x})$ are defined by section rotations of the plate about the y- and x-axes, respectively. The vertical deflection of the plate is represented by the deflection at the neutral plane of the plate denoted by $w(\mathbf{x}) \in H_0^1(\Omega)$. Thus, the vector of the displacements can be expressed as

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} 0 & z & 0 \\ 0 & 0 & z \\ 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} w \\ \beta_x \\ \beta_y \end{Bmatrix} = \mathbf{L}_u\mathbf{u} \tag{2}$$

where the basic assumption for the displacement vector of three independent field variables $\mathbf{u} \in H_0^1(\Omega) \times (H_0^1(\Omega))^2$ is

$$\mathbf{u}^{\mathrm{T}} = [w, \beta_x, \beta_y]. \tag{3}$$

In this work, the material is assumed to be linear elastic and isotropic with Young's modulus E and Poisson's ratio ν, the governing differential equations of the free vibration Reissner–Mindlin plates can be presented in a strong form as [39,40]

$$\nabla \cdot \mathbf{D}_b\boldsymbol{\kappa}(\boldsymbol{\beta}) + \lambda t\boldsymbol{\gamma} + \frac{t^3}{12}\rho\omega^2\boldsymbol{\beta} = \mathbf{0} \quad \text{in } \Omega \subset \Re^2 \tag{4}$$

$$\lambda t\nabla \cdot \boldsymbol{\gamma} + \rho t\omega^2 w = 0 \quad \text{in } \Omega \tag{5}$$

$$w = w_0; \qquad \boldsymbol{\beta} = \boldsymbol{\beta}_0 \quad \text{on } \Gamma = \partial\Omega \tag{6}$$