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The influence of boundary conditions and axial deformability on buckling behavior of two-layer composite columns with interlayer slip

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ABSTRACT

This paper presents a detailed analysis of the influence of boundary conditions and axial deformation on the critical buckling loads of the geometrically perfect elastic two-layer composite columns with interlayer slip between the layers. An investigation is based on the extension of our preliminary analytical study of slip-buckling behavior of two-layer composite columns. It is proved that the boundary conditions of composite columns with interlayer slip are interrelated in longitudinal and transverse directions. The parametric analysis reveals that the influence of different longitudinal boundary conditions on critical buckling load is significant and can be up to 20%, while, on the other hand, the influence of axial deformation is negligible.

1. Introduction

In recent years, the applications of composite layered systems in automotive, aerospace, mechanical, and structural engineering industries have increased tremendously. The main advantages of composite systems over the conventional structures are their high strength-to-weight and stiffness-to-weight ratios. However, their mechanical behavior is considerably affected by the type of the connection between the constituents. For instance, in some widely used composite structures in civil engineering, such as nailed, glued or bolted layered wood systems, wood–concrete or steel–concrete systems, an absolutely stiff connection between the layers can hardly be realized in practice. As a result an interlayer slip between the layers develops, which can, if it has a sufficient magnitude, significantly affect the mechanical behavior of the composite system.

Therefore, the interlayer slip has to be taken into consideration in what is called partial interaction analysis of composite structures. Several researches have pursued the effect of partial composite action in the analysis of the abovementioned structures, and as a result, many published papers that take into account the interlayer slip analytically or numerically are available in the literature. No attempt is made to discuss it here, but the interested reader is referred to, e.g., Adam et al. [1], Dall'Asta and Zona [2], Battini et al. [3], Čas et al. [4–6], Chen et al. [7], Silva and Sousa [8], Heuer and Adam [9], Heuer [10], Challamel [11], Ranzi and Bradford [12], Ranzi and Zona [13], Ranzi [14], Schnabl et al. [15–17], and Xu and Wu [18].

Design of structures is often based on strength and stiffness considerations. However, a structure may become unstable long before strength and stiffness criteria are violated. Therefore, buckling is an important consideration in structural design, especially when the structure is slender and lightweight. Thus, it is of practical importance to obtain the analytical solutions for such problems.

There are relatively few analytical investigations of slip-buckling problem of composite columns with interlayer slip, and to date, only a few exact models have been developed. Rassam and Goodman [19] derived a simplified solution of buckling behavior of three layered wood columns with both equal and unequal layer thicknesses. Another analytical solution of buckling problem was derived by Girhammar and Gopu [20]. An extension and generalization of the latter theory is presented in Girhammar and Pan [21]. Recent papers by Xu and Wu [18,22,23] have presented an interesting approach to the solution of slip-buckling and vibration problem of composite beam–columns when shear deformation is taken

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into account. If shear deformation is neglected, the equations for buckling load obtained by Xu and Wu [18,22,23] are the same as those presented by Girhammar and Pan [21]. The aforementioned solutions are based on what is called "second-order theory" and in Girhammar and Pan [21] also on approximate buckling length coefficients. As it is well known, this theory neglects the influence of axial deformability on the critical buckling loads. Very recently, Kryžanowski et al. [24] have proposed a slip-buckling analytical model in which the effect of axial deformability on critical buckling forces is considered while, on the other hand, the effect of shear deformation is neglected. The comparison of the critical forces with those of Girhammar and Pan [21] has shown a disagreement, which, unfortunately has not been explained in detail because only a preliminary parametric study was conducted at that time.

To complement the aforementioned studies, the main objective of the present paper is to clarify the reasons for disagreement between the results of Kryžanowski et al. [24] and those of Girhammar and Pan [21]. For this purpose, equivalently as in Kryžanowski et al. [24], a linearized stability theory is employed [25]. Hence, critical buckling forces are determined from the solution of a linear eigenvalue problem, i.e., $\det \boldsymbol{K} = 0$; see, e.g. [26].

In the numerical examples critical buckling loads are compared to those of Girhammar and Pan [21]. Based on the derived results, the reasons for the disagreement between the models are clarified. Afterwards, a parametric study is conducted in order to illustrate how the critical buckling loads of geometrically perfect two-layer composite columns are affected by axial deformability and different arrangement of end supports. In particular, it is examined, how these effects are influenced by the interlayer slip modulus, K, and column slenderness, λ.

2. Problem formulation

Consider a geometrically perfect initially straight, planar, two-layer composite column of undeformed length L. Layers, as shown in Fig. 1, are marked by letters a and b. The column is placed in the (X, Z) plane of spatial Cartesian coordinate system with coordinates (X, Y, Z) and unit base vectors $\boldsymbol{E}_X, \boldsymbol{E}_Y$ and $\boldsymbol{E}_Z = \boldsymbol{E}_X \times \boldsymbol{E}_Y$. The undeformed reference axis of the layered column is common to both layers and is defined as an intersection of the (X, Z)-plane and their contact plane. It is parametrized by the undeformed arc-length x. Local coordinate system (x, y, z) is assumed to coincide initially with spatial coordinates, and then it follows the deformation of the column. Thus, $x^a \equiv x^b \equiv x \equiv X$, $y^a \equiv y^b \equiv y \equiv Y$, and $z^a \equiv z^b \equiv z \equiv Z$ in the undeformed configuration. The two-layer composite column is loaded longitudinally at the free end by an axial conservative compressive force, P, in such way that homogeneous stress–strain state of the column at its primary configuration is achieved. For further details an interested reader is referred to, e.g., [17,24].

2.1. Kinematic equations

The deformed configurations of the reference axes of layers a and b are defined by vector-valued functions (see Fig. 1)

$$\begin{aligned}\boldsymbol{R}_0^a &= X^a\boldsymbol{E}_X + Y^a\boldsymbol{E}_Y + Z^a\boldsymbol{E}_Z = (x^a + u^a)\boldsymbol{E}_X + y^a\boldsymbol{E}_Y + w^a\boldsymbol{E}_Z,\\ \boldsymbol{R}_0^b &= X^b\boldsymbol{E}_X + Y^b\boldsymbol{E}_Y + Z^b\boldsymbol{E}_Z = (x^b + u^b)\boldsymbol{E}_X + y^b\boldsymbol{E}_Y + w^b\boldsymbol{E}_Z,\end{aligned} \tag{1}$$

in which superscripts a and b indicate that quantities are related to layers a and b, respectively. Functions u^a and w^a denote the components of the displacement vector of layer a at the reference axis with respect to the base vectors $\boldsymbol{E}_X$ and $\boldsymbol{E}_Z$. Similarly, functions u^b and w^b are related to layer b. The geometrical components u^a, w^a, u^b, and w^b of the vector-valued functions $\boldsymbol{R}_0^a$ and $\boldsymbol{R}_0^b$ are related to the deformation variables by the following equations, see, e.g. [27]:

layer a:

$$\begin{aligned}&1 + u^{a\prime} - (1+\varepsilon^a)\cos\varphi^a = 0,\\ &w^{a\prime} + (1+\varepsilon^a)\sin\varphi^a = 0,\\ &\varphi^{a\prime} - \kappa^a = 0,\end{aligned} \tag{2}$$

layer b:

$$\begin{aligned}&1 + u^{b\prime} - (1+\varepsilon^b)\cos\varphi^b = 0,\\ &w^{b\prime} + (1+\varepsilon^b)\sin\varphi^b = 0,\\ &\varphi^{b\prime} - \kappa^b = 0.\end{aligned} \tag{3}$$

Here, the prime (′) denotes the derivative with respect to x. In Eqs. (2)–(3), the deformation variables ε^a and ε^b are extensional strains; κ^a and κ^b are pseudocurvatures; while φ^a and φ^b are rotations of layers' reference axes [28].

2.2. Equilibrium equations

The composite column is subjected longitudinally to a conservative compressive force P at the free end. In addition, each layer of the two-layer composite column is subjected to interlayer contact tractions, measured per unit of layer's undeformed length, which are defined by

$$\begin{aligned}\boldsymbol{p}^a &= p_X^a\boldsymbol{E}_X + p_Z^a\boldsymbol{E}_Z = (p_t^a\cos\varphi^a + p_n^a\sin\varphi^a)\boldsymbol{E}_X\\ &\quad + (p_n^a\cos\varphi^a - p_t^a\cos\varphi^a)\boldsymbol{E}_Z,\\ \boldsymbol{p}^b &= p_X^b\boldsymbol{E}_X + p_Z^b\boldsymbol{E}_Z = (p_t^b\cos\varphi^b + p_n^b\sin\varphi^b)\boldsymbol{E}_X\\ &\quad + (p_n^b\cos\varphi^b - p_t^b\cos\varphi^b)\boldsymbol{E}_Z,\end{aligned} \tag{4}$$

where p_t^a, p_t^b, p_n^a, and p_n^b are tangential and normal components of the interlayer contact tractions, see Fig. 1. Hence, the equilibrium equations of an individual layer are, see e.g. [6,27]:

layer a:

$$\begin{aligned}&R_X^{a\prime} + p_X^a = R_X^{a\prime} + p_t^a\cos\varphi^a + p_n^a\sin\varphi^a = 0,\\ &R_Z^{a\prime} + p_Z^a = R_Z^{a\prime} - p_t^a\sin\varphi^a + p_n^a\cos\varphi^a = 0,\\ &M_Y^{a\prime} - (1+\varepsilon^a)\mathcal{Q}^a = 0,\end{aligned} \tag{5}$$

layer b:

$$\begin{aligned}&R_X^{b\prime} + p_X^b = R_X^{b\prime} + p_t^b\cos\varphi^b + p_n^b\sin\varphi^b = 0,\\ &R_Z^{b\prime} + p_Z^b = R_Z^{b\prime} - p_t^b\sin\varphi^b + p_n^b\cos\varphi^b = 0,\\ &M_Y^{b\prime} - (1+\varepsilon^b)\mathcal{Q}^b = 0,\end{aligned} \tag{6}$$

where

$$\begin{aligned}&\mathcal{N}^a = R_X^a\cos\varphi^a - R_Z^a\sin\varphi^a,\\ &\mathcal{Q}^a = R_X^a\sin\varphi^a + R_Z^a\cos\varphi^a,\\ &\mathcal{M}^a = M_Y^a,\\ &\mathcal{N}^b = R_X^b\cos\varphi^b - R_Z^b\sin\varphi^b,\\ &\mathcal{Q}^b = R_X^b\sin\varphi^b + R_Z^b\cos\varphi^b,\\ &\mathcal{M}^b = M_Y^b.\end{aligned} \tag{7}$$

R_X^a, R_Z^a, R_X^b, R_Z^b, M_Y^a, and M_Y^b in (5)–(7) represent the generalized equilibrium internal forces of a cross-section of layers a and b, respectively, with respect to the fixed coordinate basis. On the other hand, $\mathcal{N}^a$, $\mathcal{Q}^a$, $\mathcal{M}^a$, $\mathcal{N}^b$, $\mathcal{Q}^b$ and, $\mathcal{M}^a$ represent the equilibrium axial and shear internal forces and bending moments of the layers' cross-sections with respect to the rotated local coordinate system.

2.3. Boundary conditions

Kinematic equations, Eqs. (2)–(3), and equilibrium equations, Eqs. (5)–(6), constitute a system of 12 linear differential equations of the first order with constant coefficients for 12 unknown