Engineering Structures 32 (2010) 2049–2056

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

# Vulnerability assessment of single-pylon cable-stayed bridges using plastic limit analysis

D. Yan, C.C. Chang *

*Department of Civil and Environmental Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong*

ARTICLE INFO

*Article history:*
Received 20 June 2008
Received in revised form
3 March 2010
Accepted 3 March 2010
Available online 7 April 2010

*Keywords:*
Vulnerability assessment
Cable-stayed bridge
Load-carrying capacity

ABSTRACT

Vulnerability of a structure hazards can be regarded as the study of its strength and robustness against damages caused by these hazards. Measures to quantify vulnerability of a structure provide an insight into its possible performance and help to identify critical components or damage locations that can lead to a catastrophic consequence. From structural viewpoint, one possible quantitative measure for structural vulnerability assessment is the ultimate load-carrying capacity. In this study, a technique based on the plastic limit analysis is proposed for the vulnerability assessment of single-pylon cable-stayed bridges. The limit analysis assumes that, for a steady-state collapse without an inertia effect, the power done by the external forces is dissipated by the yielding components. Hence the ultimate load-carrying capacity of a bridge can be calculated directly if a proper collapse pattern is assumed. Such a concept is illustrated by assuming plastic hinge models for the healthy and the damaged bridge. The technique is illustrated on a single-pylon cable-stayed bridge with two equal side spans of 200 m. Results show that the load-carrying capacities of both the healthy and the damaged bridges can be accurately determined using this technique. The proposed technique not only provides an efficient tool to assess the vulnerability of a single-pylon cable-stayed bridge but also can be used to study the effects of different design parameters on the load-carrying capacity and the vulnerability of cable-stayed bridges.



## 1. Introduction

Since the terrorist attack on the World Trade Center in 2001, the world has been facing a growing number and intensity of such catastrophic events. Jenkins [1] described over 550 terrorist incidents worldwide against transportation targets. He concluded that terrorist attacks on public transportation has increased over the past quarter century. Bridges and related infrastructural systems are attractive terrorist targets because of their easy accessibility as well as the devastating consequence on the society when damaged. Some urgent challenges the engineers now face are to assess the vulnerability of existing civil infrastructures and to design better civil infrastructures which can provide more robust and safe performance under hazardous conditions.

Vulnerability of a structure under manmade or environmental hazards can be regarded as the study of its strength and robustness against damages caused by these hazards. A structure is vulnerable if a relatively small damage leads to a disproportionately large consequence [2]. Measures to quantify vulnerability of a structure can help to gain a better insight into its possible performance and furthermore to identify critical components or damage locations that can result in a catastrophic consequence. Williamson and Winget [3] summarized some research directions and recommended a cost-effective framework to investigate the performance of bridges under terrorist attack. The proposed framework included threat identification, vulnerability assessment, emergency response and countermeasures.

To study the vulnerability of structures, Agarwal et al. [2] proposed a deterministic theory based on the concepts of structural form and connectivity. A measure of well-formedness and clustering procedure was developed to identify failure scenarios. Schafer and Bajpai [4] extended a seismic performance-based design theory to study the vulnerability of frame structures. In their study, the vulnerability of frames was defined as the degradation of their buckling loads due to a sudden removal of some structural members. Ettouney et al. [5] used a nonlinear finite element analysis to simulate the progressive failure of a frame structure under natural and man-made hazards. Recently, de Felice [6] presented an approach using nonlinear beam elements with fibre cross-section for modeling the ultimate behavior of multi-span masonry arch bridges. While the finite element analysis could emulate the progressive failure and even the collapse of structures under various hazardous scenarios, the analysis was rather time consuming and could not be used effectively for a conceptual or detailed design purpose. There is a need to develop some approximate methods which can be used to assess the vulnerability of structures under extreme conditions.

* Corresponding author. Tel.: +852 2719 8354.
*E-mail addresses:* ydxaa@ust.hk (D. Yan), cechang@ust.hk (C.C. Chang).

0141-0296/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2010.03.005

The limit analysis has been proposed by several researchers to obtain the ultimate load-carrying capacity of some structures such as frames [7], thin-walled beams and columns [8], and masonry structures [9–11]. The analysis provided a means to directly assess the ultimate capacity of a structure without analyzing the entire history of response. A comprehensive overview of the limit analysis was summarized by Jirásek and Bažant [12] for beam and frame structures. Also, pushover analyses were conducted by Bignell et al. [13] to assess the seismic vulnerability of wall pier supported highway bridges. In this study, a technique based on the plastic limit analysis is proposed for the vulnerability assessment of single-pylon cable-stayed bridges. The limit analysis assumes that, for a steady-state collapse without an inertia effect, the power done by the external forces should be dissipated by the yielding components. Hence the ultimate load-carrying capacity of a bridge can be calculated directly if a proper collapse pattern can be assumed. Such a concept is illustrated by assuming plastic hinge models for the healthy and the damaged bridge. The technique is illustrated on a single-pylon cable-stayed bridge with two equal side spans.

## 2. Vulnerability of bridges

In this study, it is assumed that the act of terrorism damages the bridge such that a sudden fracture appears at some location on the bridge girder. The structural safety of the bridge before and after damage is quantitatively reflected through its ultimate load-carrying capacity.

It is also assumed that all bridge components have elastic-perfectly plastic behavior. Denote the uniformly distributive design load (including both the dead load and the live load) that acts on the bridge deck as $q$. The ultimate load-carrying capacities of the healthy and the damaged bridge can be expressed as $N_0q$ and $N_dq$ where $N_0$ and $N_d$ are the ultimate load multipliers for the healthy and the damaged bridge, respectively. The vulnerability of a cable-stayed bridge can be quantitatively assessed through the following index:

$$V = \frac{N_0 - N_d}{N_0}. \tag{1}$$

Note that the defined vulnerability index $V$ varies between 0 and 1. Physically, $V = 0$ means that the damage of bridge deck has no effect on the bridge's ultimate load-carrying capacity while $V = 1$ represents that the damage of bridge deck leads to the complete collapse of the bridge. A higher vulnerability index represents that the overall safety of the bridge is more significantly affected by the damage.

The ultimate load-carrying capacity of a cable-stayed bridge, whether it is healthy or damaged, can be obtained via a nonlinear static finite element analysis in which the applied load is incrementally increased until the instantaneous global stiffness matrix of the bridge becomes singular. Such a calculation however is rather time consuming. In the following, an approximate method based on the plastic limit analysis is proposed for estimating the load-carrying capacity of the healthy and the damaged bridges. To simplify the analysis, the cable-stayed bridge considered in this study is assumed to have only one single pylon and is symmetrical along the span-wise direction. Hence, both the healthy and the damage bridge can be analyzed using two-dimensional finite element models.

## 3. Ultimate load-carrying capacity of the healthy bridge

Fig. 1(a) shows a single-pylon cable-stayed bridge with a back span length of $L_b$, a main span length of $L_m$ and a pylon height of $H$. The back span and the main span are supported by $n_b$ and $n_m$ stayed cables, respectively. The bridge deck is assumed to be hinged at one end and roller supported at the other. The girder is also roller supported by the pylon. For convenience, two coordinate systems, $x - y$ and $x_p - y_p$, are chosen for the girder and the pylon, respectively. Fig. 1(b) shows an elastic deformation shape of the bridge under a uniform distributive load of $Nq$ where $N$ is a load multiplier. As compared to the incremental analysis mentioned above, the plastic limit analysis is an alternative approach to determine the ultimate load-carrying capacity of a structure. The limit analysis assumes the formation of plastic hinges at locations where the bending moment reaches local extrema. Fig. 1(c) shows a possible plastic hinge model for the bridge. A total of five plastic hinges are assumed in this model: one at the pylon base, two in the back span located at $L_1$ and $L_1 + L_2$ from the left end and two in the main span located at $L_4$ and $L_3 + L_4$ from the right end. The corresponding rotational angles are denoted as $\theta_i$, $i = 1$ to 5. Note that the bridge would collapse when all the bridge cables anchored to the plastic hinge region $L_1 + L_2$ or $L_3 + L_4$ yield. During a steady-state collapse without the inertia effect, the power done by the external forces $\dot{W}$ should be dissipated by the yielding components thus leads to the power equality [12],

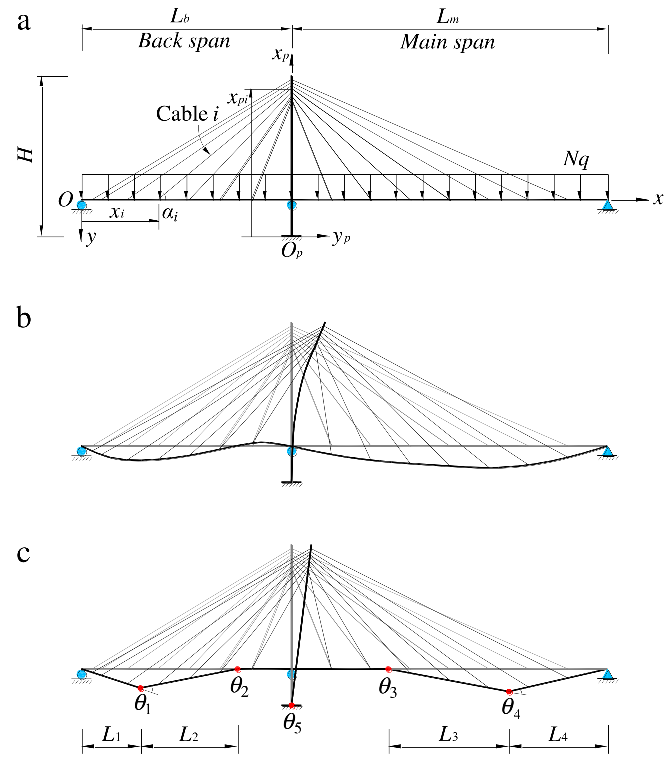


**Fig. 1.** (a) A healthy single-pylon cable-stayed bridge under a uniform distributive load $Nq$; (b) its elastic deformation shape; and (c) its ultimate deformation shape approximated by a plastic hinge model.

$$\dot{W} = \dot{D} \tag{2}$$

where $\dot{D}$ is the rate of energy dissipation. Alternatively, an equation can also be formulated using the principle of virtual work,

$$\delta W = \delta D, \tag{3}$$

where $\delta W$ and $\delta D$ are the virtual work done by the external forces and the internal forces, respectively. Based the plastic hinge model shown in Fig. 1(c), $\delta D$ can be expressed as

$$\delta D = \sum_{i=1}^{n_b+n_m} f_{c_i} A_{c_i} \cdot |\delta e_i| + f_g Z_g \cdot (|\delta\theta_1| + |\delta\theta_2| + |\delta\theta_3| + |\delta\theta_4|) + f_p Z_p \cdot |\delta\theta_5|, \tag{4}$$