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Simplified evaluation of the vibration period and seismic response of gravity dam–water systems

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ABSTRACT

This paper proposes a practical procedure for a simplified evaluation of the fundamental vibration period of dam–water systems, and corresponding added damping, force and mass, all key parameters to assess the seismic behavior. The proposed technique includes the effects of dam geometry and flexibility, dam–reservoir interaction, water compressibility and varying reservoir level. The mathematical derivations of the method are provided considering both incompressible and compressible water assumptions. In the former case, we propose a closed-form expression for the fundamental vibration period of a dam–reservoir system. When water compressibility is included, we show that the fundamental vibration period can be obtained by simply solving a cubic equation. The proposed procedure is validated against classical Westergaard added mass formulation as well as other more advanced analytical and finite element techniques. Gravity dam monoliths with various geometries and rigidities impounding reservoirs with different heights are investigated. The new approach yields results in excellent agreement with those obtained when the reservoir is modeled analytically, or numerically using potential-based finite elements. The analytical expressions developed and the procedure steps are presented in a manner so that calculations can be easily implemented in a spreadsheet or program for simplified and practical seismic analysis of gravity dams.

1. Introduction

Considering the effects of fluid–structure dynamic interactions is important for the design and safety evaluation of earthquake-excited gravity dams. Significant research has been devoted to this subject since the pioneering work of Westergaard [1] who modeled hydrodynamic loads as an added mass attached to the dam upstream face. Although Westergaard's analytical formulation was developed assuming a rigid dam impounding incompressible water, it has been widely used for many decades to design earthquake-resistant concrete dams because of its simplicity. During the last four decades, several researchers developed advanced analytical and numerical approaches to account for dam deformability and water compressibility in the seismic response of concrete dams [2–12]. Most of these methods are based on a coupled field solution through sub-structuring of the dam–reservoir system, making use of analytical formulations, finite elements, boundary elements or a mix of these techniques. In the approach proposed by Chopra and collaborators [2–4,7], the reservoir is modeled analytically as a continuum fluid region extending towards infinity in the upstream direction. When finite or boundary elements are used, the reservoir has to be truncated at a finite distance and appropriate transmitting boundary conditions have to be applied at the cutting boundaries to prevent reflection of spurious waves as discussed by the authors in a previous work [13]. Some procedures were implemented in numerical codes specialized in two- and three-dimensional analyses of concrete dams [9,14], and some were validated against experimental findings from in situ forced-vibration tests [15–18]. Although such sophisticated techniques were proven to efficiently handle many aspects of dam–reservoir interactions, their use requires appropriate expertise and specialized software. For practical engineering applications, simplified procedures are still needed to globally evaluate the seismic response of gravity dams, namely for preliminary design or safety evaluation purposes [19–21].

The fundamental vibration period of dam–reservoir systems is a key factor in the assessment of their dynamic or seismic behavior. Most seismic provisions and simplified procedures use the fundamental vibration period as an input parameter to determine seismic design accelerations and forces from a site-specific earthquake response spectrum. It is therefore crucial to dispose of accurate and yet practical expressions to evaluate the fundamental period of gravity dams dynamically interacting with their impounded reservoirs. Hatanaka [22] developed simplified expressions to estimate the fundamental vibration period of dams with empty reservoirs. He approximated the dam geometry

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Nomenclature

Abbreviations

ESDOF Equivalent single degree of freedom.
FRF Frequency response function.

Roman symbols

A_1, A_2, A_3, A_4 Coefficients given by Eqs. (59)–(63).
a_1, a_2, a_3 Coefficients used for cubic approximation of structural mode shapes.
B_0, B_1 Hydrodynamic parameters given by Eqs. (22) and (23), respectively.
B_{0n}, B_{1n} Hydrodynamic parameters given by Eqs. (24) and (25), respectively.
$\widehat{B}_{0n}, \widehat{B}_{1n}$ Hydrodynamic parameters given by Eqs. (32) and (33), respectively.
$C_n, \widetilde{C}_n$ nth generalized damping of the dam and dam–reservoir system, respectively.
C_r Velocity of pressure waves in the reservoir.
D_1, D_2 Coefficients given by Eq. (65).
E_s Modulus of elasticity of the dam.
F_{st} Total hydrostatic force exerted on dam upstream face.
F_n, G_n Functions given by Eq. (34).
f_1 Equivalent lateral force given by Eq. (80).
f_{sc} Equivalent lateral force including higher mode effects as given by Eq. (83).
H_r, H_s Reservoir and dam heights, respectively.
I_{jn} Integral given by Eq. (8).
K_1 Generalized stiffness of the dam at fundamental vibration mode.
$L_n, \widetilde{L}_n$ nth generalized forces of the dam and dam–reservoir system, respectively.
$\mathbf{M}$ Mass matrix of the dam monolith.
M_s Total mass of the dam monolith.
m_i Westergaard added mass at node i of the dam finite element mesh.
$M_n, \widetilde{M}_n$ nth generalized masses of the dam and dam–reservoir system, respectively.
N_r, N_s Number of considered reservoir and structural modes, respectively.
$\bar{\mathbf{Q}}, \bar{Q}_n$ Vector in Eq. (11) and its elements given by Eq. (13), respectively.
$p, \bar{p}$ Hydrodynamic pressure and corresponding FRF, respectively.
$\bar{p}_0, \bar{p}_j$ Hydrodynamic pressure FRFs given by Eq. (3).
$\bar{p}_{0n}, \bar{p}_{jn}$ Hydrodynamic pressure FRFs given by Eqs. (4) and (5), respectively.
$\widehat{\bar{p}}_0$ Real-valued hydrodynamic pressure given by Eq. (84).
R_1, R_r Frequency ratios given by ω_1/ω_0 and ω_r/ω_0, respectively.
$\bar{\mathbf{S}}, \bar{S}_{nj}$ Matrix in Eq. (11) and its elements given by Eq. (12), respectively.
S_a Pseudo-acceleration ordinate of the earthquake design spectrum.
t Time.
T_1, T_r Fundamental periods of the dam and dam–reservoir system, respectively.
U coefficient given by Eq. (67).
$\bar{u}, \bar{\ddot{u}}$ FRFs for horizontal displacement and acceleration, respectively.
V Coefficient given by Eq. (67).
V_i Volume of water tributary to node i of the dam finite element mesh.
$\bar{v}, \bar{\ddot{v}}$ FRFs for vertical displacement and acceleration, respectively.
$\ddot{x}_g, \ddot{x}_g^{(\max)}$ Ground acceleration time history and peak ground acceleration, respectively.
y_i Height of node i of the dam finite element mesh.
$\bar{\mathbf{Z}}, \bar{Z}_j$ Vector of generalized coordinates and jth generalized coordinate, respectively.

Greek symbols

$\gamma_i, \widehat{\gamma}_i$ Coefficients given in Table 1 for $i = 1 \ldots 6$.
Γ Variable given by Eq. (65).
$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$ Analytical solutions of Eq. (64) as given by Eq. (66).
Γ^* Real solution of Eq. (64).
Δ Discriminant of Eq. (64).
δ_{nj} Kronecker symbol.
ε Error estimator.
$\zeta_i, \widehat{\zeta}_i$ Coefficients given in Table 2 for $i = 1 \ldots 3$.
η Ratio of reservoir level to dam height, i.e. H_r/H_s.
$\theta, \widehat{\theta}, \Theta$ Parameters given by Eqs. (76), (43) and (42), respectively.
κ_n Function given by Eq. (7).
λ_n nth reservoir eigenvalue.
μ_s Mass of the dam per unit height.
ν Poisson's ratio of dam concrete.
ξ_n nth fraction of critical damping of the dam.
$\widetilde{\xi}_r$ Equivalent damping ratio of the dam–reservoir ESDOF system.
ρ_r, ρ_s mass densities of water and dam concrete, respectively.
τ Coefficient given by Eq. (67).
$\varphi, \widehat{\varphi}, \Phi$ Parameters given by Eqs. (57), (39) and (38), respectively.
χ Frequency parameter defined by R_r^2.
$\boldsymbol{\psi}_n, \psi_j^{(x)}$ nth structural mode shape and x-component of the jth structural mode shape.
ω Exciting frequency.
ω_0 Fundamental vibration frequency of the full reservoir.
ω_n nth vibration frequency of the dam.
ω_r Fundamental vibration frequency of the dam–reservoir system.

as a symmetrical triangle and distinguished the cases where bending or shear effects are predominant in the dynamic response of the dam. Considering analogy with beam theory, Okamoto [23] proposed simplified formulas to estimate the fundamental vibration periods of dams with empty and full reservoirs. Chopra [2,4] analyzed several idealized triangular dam cross-sections to obtain an approximate fundamental vibration period and corresponding mode shape of typical gravity dams with an empty reservoir. These standard dynamic properties and related quantities were implemented in simplified earthquake response analyses of gravity dams [19,20]. To determine the fundamental vibration period of a dam including impounded water effects, Chopra and collaborators [2–4,7,15] first obtained the frequency response curves characterizing dam–reservoir vibrations, and then identified the fundamental vibration frequency as the one corresponding to the first resonance on the curves. The authors found that hydrodynamic effects lengthen the fundamental vibration period of gravity dams and the results obtained for