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The influence of a train's critical speed and rail discontinuity on the dynamic behavior of single-span steel bridges

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ABSTRACT

In this paper, the influence of train speed on the dynamic behavior of lightweight steel bridges is studied. Moreover, an existing rail discontinuity is also considered and its combined effect on the dynamic behavior of the bridge is taken into account. In particular, this paper studies first the effect of the axis frequency on the bridge dynamic behavior as trains enter into the bridge span, which is coincident or almost coincident with the fundamental eigenfrequency of the bridge. This entrance frequency of the wheels obviously depends on the axis distance and the train's speed. Secondly, a rail discontinuity is considered and its combined effect on the dynamic behavior of the bridge due to the developing impact forces is studied. The theoretical formulation presented herein is based on a continuous approach and the equations of motion of the bridge are determined through the use of a simple two-degrees-of-freedom model.

1. Introduction

A lot of research work has been reported during the last 100 years dealing with the dynamic response of railway bridges, and later of highway bridges, under the influence of moving loads. Extensive literature references on this subject can be found in the excellent book of Frýba [1].

Two early interesting contributions in this area exist thanks to Stokes [2] and Zimmerman [3]. In 1905, Krýlov [4] presented a complete solution to the problem of the dynamic behavior of a prismatic bar subjected to a load with constant magnitude and moving with a constant velocity. In 1922, Timoshenko [5] solved the same problem but for a harmonic pulsating moving force. Another pioneering work on this subject was presented in 1934 by Inglis [6], in which numerous parameters were taken into account. Finally, in 1951, Hillerborg [7] presented an analytical solution to the above problem by means of the Fourier method.

Despite the availability of powerful computers, most of the methods employed today for analyzing bridge vibration problems are essentially based on Inglis's or Hillerborg's early techniques. Relevant publications are those of Saller [8], Jeffcot [9], Steuding [10], Honda et al. [11], Gillespi [12], Green and Cebon [13], Green et al. [14], Zibdeh and Reckwitz [15], Lee [16], Michaltsos et al. [17], Xu and Genin [18], Foda and Abduljabbar [19], and Michaltsos [20,21].

On the other hand, in engineering practice, and despite the large number of studies for over 50 years, bridges have been designed to account for dynamic loads by increasing the design live loads by a semi-empirical "impact factor" or "dynamic load allowance", applied also to other structural systems subjected to dynamic loads.

Recently, many research programs in different countries have been focused on the effect of the characteristics of a bridge or a vehicle on the dynamic response of the bridge. One can mention the relevant programs in the USA [22], in the UK and Canada [23], in countries belonging to the Organization for Economic Cooperation and Development (OECD) [24], in Switzerland [25], etc.

Although there are many important publications in this field, one must especially refer to the important experimental research by Cantieri [26], who studied several models of moving loads.

The dynamic response of railway bridges subjected to moving loads is one of the fundamental engineering problems requiring design and maintenance solutions. Due to the design characteristics of train wagons and carriage coaches (see Fig. 1), lateral forces also develop, producing secondary oscillations that are, in general, negligible, though under very special conditions, these secondary oscillations may have a significant effect.

There are numerous publications on the dynamic response of bridges, dealing with phenomena produced by single moving loads or by trains moving with normal or high speeds. One must refer to the works of Matsuura [27], Diana and Cheli [28], Frýba [29], Yang et al. [30], De Roeck et al. [31], Gao and Pan [32], Xia et al. [33], and Ivanchenko [34,35]. Finally, Steenbergen [36] has contributed

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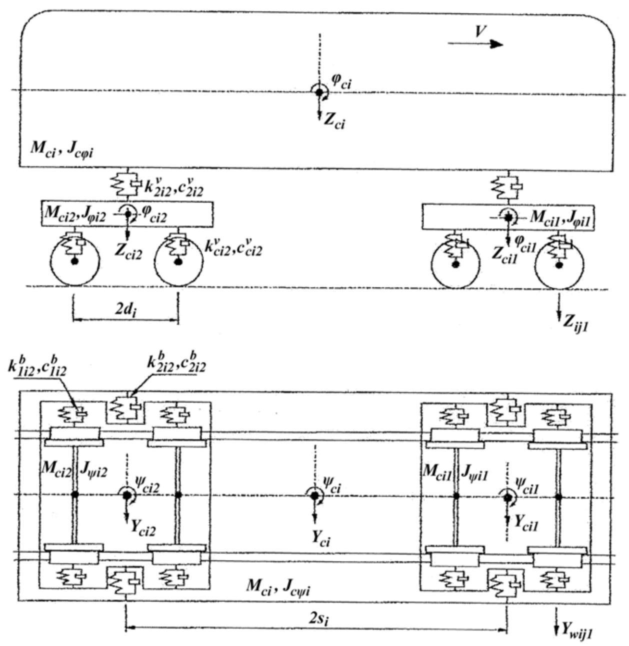

Fig. 1. Train wagon geometrical and mechanical characteristics.

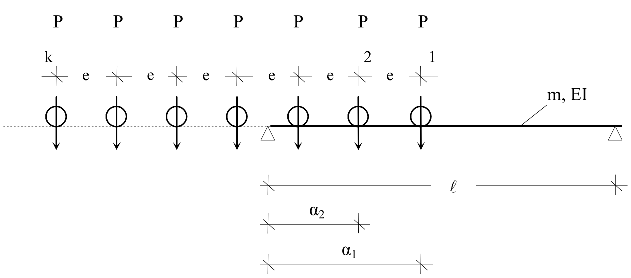

Fig. 2. Model of a train crossing a single-span bridge.

in the area of dynamic wheel–rail contact analysis by proposing suitable models for modeling wheels and rail discontinuities.

For long trains, the axis frequency entering the bridge depends obviously on the axis distance and the train's speed. Moreover, although contemporary rails are made without a joint-gap, there are many cases in which the designer is obliged to consider the existence of, or to construct, a discontinuity in the rails.

In this work, the effect of the above two parameters on the dynamic behavior of a bridge is thoroughly examined, i.e. the effect of the axis frequency on the bridge dynamic behavior, as trains enter into the bridge span, which is coincident or almost coincident with the fundamental eigenfrequency of the bridge as well as the effect of an existing discontinuity of the rails.

A simple two-degrees-of-freedom model is established for the solution of the bridge vibration problem, while the theoretical formulation is based on a continuum approach, which has been widely used in the literature for analyzing such bridge types.

2. Theoretical analysis

Let us consider now a single-span bridge with length ℓ (Fig. 2), made from isotropic and homogeneous material with modulus of elasticity E, mass per unit length m, and moment of inertia I, that is crossed by a train consisting by k wagons or $2k$ axes, each being at distance e from each other, and carrying load P moving with constant velocity υ. Let us also consider that at point $x = b$ a discontinuity in the rails exists.

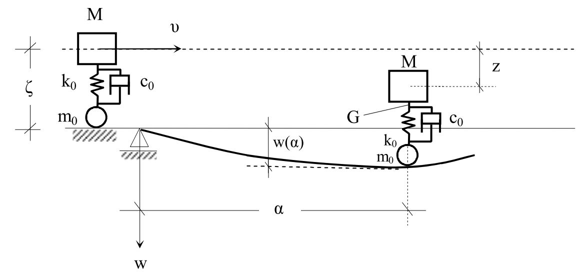

Fig. 3. Single axis load model.

Using Dirac's delta function and Heaviside's unit-step one, the equation of motion of the bridge becomes

$$\begin{aligned}
EIw'''' + c\dot{w} + m\ddot{w} &= P\delta(x-a) \\
&\times \sum_{\rho=1}^{k}\left[H\left(t-\frac{(\rho-1)e_1}{\upsilon}\right) - H\left(t-\frac{\ell+(\rho-1)e_1}{\upsilon}\right)\right] \\
&+ P_{\text{imp}}\delta(x-b)\left[\sum_{\rho=1}^{k}\delta\left(t-\frac{b+(\rho-1)e_1}{\upsilon}\right)\right. \\
&\left. + \sum_{\rho=1}^{k}\delta\left(t-\frac{(b+e_2)+(\rho-1)e_1}{\upsilon}\right)\right] \qquad (1)
\end{aligned}$$

where time t is valid in the interval from $t = 0$ to $t = [\ell + (k-1)e]/\upsilon$ and P_{imp} is the impact force developed by the rail discontinuity.

2.1. The load of a single axis

Considering an arbitrary axis of the train crossing the bridge (see Fig. 3), we have

$$P = M(g - \ddot{z}) + m_o(g - \ddot{w}). \qquad (2a)$$

Making a cut at point G (Fig. 3), we obtain

$$M\ddot{z} = -k_o(z-w) - c_o(\dot{z} - \dot{w})$$

or finally

$$\left.\begin{aligned}
&\ddot{z} + 2\beta_o\dot{z} + \gamma_o^2 z = \gamma_o^2 w + 2\beta_o w \\
&\text{where } \beta_o = \frac{c_o}{2M}, \quad \text{and} \quad \gamma_o^2 = \frac{k_o}{M}.
\end{aligned}\right\} \qquad (2b)$$

The solution of Eq. (2b) is

$$\left.\begin{aligned}
&z(t) = \frac{1}{\omega_o}\int_0^t e^{-\beta_o(t-\tau)}\left[\gamma_o^2 w(x,\tau) + 2\beta_o\dot{w}(x,\tau)\right] \\
&\quad \times \sin\omega_o(t-\tau)\,d\tau \\
&\text{with } \omega_o = \sqrt{\gamma_o^2 - \beta_o^2}.
\end{aligned}\right\} \qquad (3a)$$

According to Leibnitz's formula that for the expression $G(x) = \int_0^x Q(x,t)dt$, it holds that $\frac{dG(x)}{dx} = Q(x,x) + \int_0^x \frac{\partial(x,t)}{\partial x}dt$, from Eq. (3a) we obtain

$$\begin{aligned}
\dot{z}(t) = &\frac{1}{\omega_o}\int_0^t e^{-\beta_o(t-\tau)}\left[\gamma_o^2 w(x,\tau) + 2\beta_o\dot{w}(x,\tau)\right] \\
&\times \left[\omega_o\cos\omega_o(t-\tau) - \beta_o\sin\omega_o(t-\tau)\right]d\tau. \qquad (3b)
\end{aligned}$$

Thus, Eq. (2a) for load P, because of Eqs. (3a) and (3b), is given by

$$\begin{aligned}
P(x,t) &= (M+m_o)g - m\ddot{w} - M\ddot{z} \\
&= (M+m_o)g - m\ddot{w} + k_o(z-w) + c_o(\dot{z}-\dot{w})
\end{aligned}$$