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Limit and shakedown analysis of 3D steel frames via approximate ellipsoidal yield surfaces

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ABSTRACT

This work concerns finite element limit and shakedown analysis of spatial steel frames under nonlinear yield criteria of steel sections. Inner and outer ellipsoidal approximations to the plastic interaction yield surfaces are systematically constructed. Under ellipsoidal yield criteria, the arising computational optimization problem becomes a second-order cone programming problem, for which free and commercial software packages are available, capable of treating large-scale problems. The present study is focused on approximations to the nonlinear plastic interaction provisions of Eurocode 3. Moreover, a well-known criterion due to Orbison is considered. Examples of limit and shakedown analysis of spatial frames under the aforementioned ellipsoidal approximations are presented and several aspects are discussed. For comparison, a fairly general interior point algorithm is also successfully applied to the limit and shakedown analysis under the original, non-ellipsoidal form of the Orbison criterion.

1. Introduction

Eurocode 3 (EC3) and other structural steel design codes permit the application of plastic analysis under the condition that the member sections are compact (in the EC3 terminology [1], they are categorized as Class 1 sections). Plastic analysis can be performed either via a step by step method or via a limit analysis technique. Limit analysis (LA) concerns one-parametric monotonic loading histories, while its generalization, shakedown analysis (SDA), concerns arbitrary loading histories within given bounds (see e.g. [2–4]). LA can treat second-order effects, but SDA has severe difficulties concerning this aspect. The numerical treatment of LA/SDA is mainly performed by coupling finite element methods (FEM) with mathematical programming techniques.

If the involved yield criteria are linear or piecewise linearized, the optimization problem becomes a linear programming (LP) problem. Otherwise, a nonlinear programming (NLP) problem must be solved. In years 1970–1990, the Simplex algorithm of LP was the single general computational technique available [5,6]. Simplex techniques can effectively treat very large problems, which is crucial for LA/SDA. This way, the LP line of research has been proven very fruitful (cf. e.g. [7–11]). Examples of recent relevant achievements can be found in [12–14]. LA/SDA under nonlinear yield criteria have been addressed by several researchers in the last fifteen years (cf. e.g. [15–20]).

Simplex algorithms belong to the so-called active-set methods. Since the 1980s, the mathematical programming research community has developed a general alternative to the active-set methods: the so-called Interior Point Methods (IPMs), firstly applied to LP problems and subsequently extended to NLP situations. Today, IPMs can treat large sparse problems of several optimization problem classes [6,21]. The so-called second-order cone programming (SOCP) problems are a generalization of LP problems, where a linear function is to be minimized, subjected to linear equality conditions and quadratic/conic inequality constraints. In turn, SOCP is generalized to semidefinite programming (SDP) problems. Reliable and effective software for all these problem classes is available [22,23]. In the LA/SDA context, several researchers have either developed specific IPM algorithmic techniques or explored existing software (cf. e.g. [24–33]).

To the authors' knowledge, research reports on LA/SDA of steel frames under non-linearized yield criteria are rare. In our opinion, this can mainly be attributed to two reasons: (a) to the shape diversity of the steel section nonlinear yield surfaces proposed in the literature, which prevents the effective application of general NLP techniques and (b) to the importance of local softening, caused e.g. by local buckling, lateral-torsional buckling or by the semi-rigid nature of some steel connection types. Consequently, a linearization of the yield surfaces is generally preferred. A min–max optimization technique for frames under nonlinear yield criteria has been developed in [34]. An iterative method for reinforced concrete frames is presented in [35] using successive linearization of the section criteria, which can be extended to steel frames. The extensive computer package CEPAO, developed in Liège [36], is a recent example of LP plastic analysis techniques for 3D steel frames.

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Limit state analyses of strain softening frames have been studied in a series of works (cf. e.g. [37,38] and the references cited therein). Interestingly, softening branches lead to nonconvex optimization problems with complementarity/equilibrium constraints, a mathematical programming problem class which, today, is mainly encountered by IPM techniques.

Ellipsoids are convex, smooth and represent one of the simplest mathematical vector norms. Ellipsoidal yield criteria, such as von Mises and Hill criteria, possess a prominent position in plasticity. LA and SDA under these criteria lead to a SOCP problem (cf. e.g. [26,29]). Ellipsoidal yield criteria have been applied for the limit analysis of geotechnical and composite materials [39,40]. Recently, enclosing ellipsoids have been used in multiaxial fatigue research [41,42].

The present study addresses the application of IPM techniques to the finite element limit and shakedown analysis of 3D steel frames without geometric nonlinearity effects and softening. Inner and outer ellipsoidal approximations (bounding ellipsoids) to the steel section yield criteria are constructed. This way, LA/SDA lead to a SOCP problem, for which reliable software exists, capable of treating large-scale problems. Particular attention is paid to the EC3 plastic interaction surfaces. Moreover, the Orbison yield criterion [43] for wide flange shapes, used by other researchers in nonlinear frame analyses [44,45], is included. For comparison purposes, a general IPM algorithm is also applied for the original Orbison criterion.

Computational treatments of LA/SDA can be classified into two main groups: (a) the embedding approach and (b) the interfacial approach. In the embedding approach, the solution of the optimization problem is incorporated in a single nonlinear FEM code. In general, this code is highly efficient, but non-accessible by other researchers. The interfacial approach contains two steps and it is characterized by a clear interface between FEM and mathematical programming. In the first step, a linear FEM code is used to produce the necessary data. In the second step, these data, along with the description of the plastic interaction surfaces, feed an available optimization package. The interfacial approach, followed in this work, can be easily applied by non-experienced engineers, giving the possibility to exploit several existing optimization codes.

The plan of the paper is as follows. Section 2 presents the formalism used for nonlinear yield criteria of compact steel sections. Section 3 contains the addressed problems of the FEM-discretized limit and shakedown analysis. Section 4 describes the construction of the approximating ellipsoids and Section 5 contains LA/SDA examples. Concluding remarks in Section 6 close the paper.

Notation: Regular letters are used for scalars and standard vector-matrix notation is adopted. $|a|$ is the absolute value of scalar a and $\|\mathbf{x}\|$ denotes the Euclidean length of vector $\mathbf{x}$. The determinant of matrix $\mathbf{X}$ is $\det(\mathbf{X})$. Vectorial inequalities hold component wise. Upper calligraphic letters denote sets. We define the 3D unit sphere $\mathcal{S}$ and the 3D unit cube $\mathcal{Q}$ by:

$$\mathcal{S} = \{\mathbf{x} \in R^3 : \|\mathbf{x}\| \le 1\},$$
$$\mathcal{Q} = \{\mathbf{x} \in R^3 : -1 \le x_k \le 1, k = 1, 2, 3\}. \tag{1}$$

2. Nonlinear yield criteria of compact steel sections

Let us consider a spatial steel frame Ω, whose members are constructed from doubly-symmetric, compact sections. Let $\tilde{\mathbf{s}}_j = (N, V_y, V_z, M_t, M_y, M_z)^T$ be the standard 6-dimensional local vector containing the axial/shearing forces and twisting/bending moments which act at the j-th section. The dimensionless local stress-resultant vector $\mathbf{s}_j$ is given by:

$$\mathbf{s}_j = (n, v_y, v_z, m_t, m_y, m_z)^T,$$
$$\mathbf{s}_j = \mathbf{N}_j^{-1}\,\tilde{\mathbf{s}}_j, \quad \tilde{\mathbf{s}}_j = \mathbf{N}_j\,\mathbf{s}_j \tag{2}$$

where $\mathbf{N}_j$ is the 6×6 diagonal matrix with diagonal entries equal to the respective individual plastic capacities N_{pl}, $V_{y.pl}$, $V_{z.pl}$, $M_{t.pl}$, $M_{y.pl}$ and $M_{z.pl}$. For $m_y - m_z - n$ plastic interactions considered in this work, the section yield criteria read:

$$\mathbf{z}_j \in \mathcal{Q}_j^I, \qquad \bar{\mathbf{z}}_j \in \mathcal{Q}_j^N \tag{3}$$
$$\mathbf{z}_j \in \mathcal{F}_j \tag{4}$$

where $\mathbf{z}_j$ is the subvector of $\mathbf{s}_j$, collecting the terms entering the plastic interaction conditions and $\bar{\mathbf{z}}_j$ is the remaining subvector of $\mathbf{s}_j$:

$$\mathbf{z}_j = (m_y, m_z, n)^T = \mathbf{Z}_j\,\mathbf{s}_j,$$
$$\bar{\mathbf{z}}_j = (v_y, v_z, m_t)^T = \bar{\mathbf{Z}}_j\,\mathbf{s}_j$$

with

$$\mathbf{Z}_j = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$
$$\bar{\mathbf{Z}}_j = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \tag{5}$$

The 3D cubes in Eq. (3) describe the bounds posed to the individual plastic capacities. These cubes, defined by Eq. (1), can be locally modified in order to take into account different tension/compression capacities. Eq. (4) defines the plastic interaction relations and set $\mathcal{F}_j$ is described by:

$$\mathcal{F}_j = \{\mathbf{z} \in R^3 : f(\mathbf{z}) \le 1\}. \tag{6}$$

Several types of sets $\mathcal{F}_j$ with linear or nonlinear functions in Eq. (6) have been studied in the literature and incorporated in structural steel design codes. Fig. 1 depicts the plastic interaction surfaces discussed in this work. Fig. 1a shows the rhombic criterion, 1b shows the Orbison criterion and 1c the EC3 nonlinear criterion for rolled or welded H- and I-sections. The EC3 criteria for circular (CHS), rectangular (RHS) and quadratic (QHS) hollow sections are depicted in Fig. 1d–f. EC3 permits, alternatively, the use of the rhombic criterion, which reads:

$$\mathcal{R} = \{\mathbf{z} \in R^3 : |n| + |m_y| + |m_z| \le 1\}. \tag{7}$$

The Orbison criterion [43] is given by:

$$f(\mathbf{z}) = 1.15\,n^2 + m_y^2 + m_z^4 + 3.67\,n^2\,m_y^2 + 3.0\,n^6\,m_z^2 + 4.65\,m_y^4\,m_z^2. \tag{8}$$

The nonlinear plastic interaction surfaces of EC3 are based on the function:

$$f(\mathbf{z}) = \left[\frac{|m_y|}{c_y(n)}\right]^{\alpha(n)} + \left[\frac{|m_z|}{c_z(n)}\right]^{\beta(n)} \tag{9}$$

and its specific form depends on the particular section type with eventual use of section parameters (see Fig. 2 for the definition of the involved geometry).

Eq. (9) is specialized to H- and I-sections (Fig. 1c) via the functions:

$$\alpha(n) = 2, \qquad \beta(n) = \max[\,1,\ 5\,|n|\,],$$
$$c_y(n) = \min\left[\,1,\ \frac{1-|n|}{1-0.5a_s}\,\right],$$
$$c_z(n) = 1 \quad \text{if } |n| \ge a_s,$$
$$c_z(n) = 1 - \left(\frac{|n|-a_s}{1-a_s}\right)^2, \quad \text{otherwise.}$$

Section parameter a_s is given by the next formula, where A denotes section area:

$$a_s = \min\left[\,0.5,\ \frac{A - 2bt_f}{A}\,\right].$$