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Coupled lateral–torsional frequencies of asymmetric, three-dimensional structures comprising shear-wall and core assemblies with stepwise variable cross-section

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ABSTRACT

A simple and accurate model for asymmetric, three-dimensional wall-core structures is developed that enables any desired natural frequency to be determined by a method which guarantees that no natural frequencies can be missed. The model assumes that the primary walls and cores run in two orthogonal directions and that their properties may vary in a stepwise fashion at one or more storey levels. A vectorial approach is used to generate the governing differential equations for coupled flexural-torsional motion that are finally incorporated into an exact dynamic stiffness matrix (exact finite element) that can model any uniform segment of the structure. A model of the original structure can then be assembled in the usual way. Since the mass of each segment is assumed to be uniformly distributed, it is necessary to solve a transcendental eigenvalue problem, which is accomplished using the Wittrick–Williams algorithm. When the structure can be represented realistically by a uniform cantilever, solutions can be found easily, by hand. A parametric study comprising five, three-dimensional, asymmetric wall-core structures is given to compare the accuracy of the current approach with that of a full finite element analysis.

1. Introduction

The type of mathematical model developed herein can be classified as a simplified global model. This implies that the original structure is treated holistically, but simplified prior to analysis so that only its dominant characteristics are retained. The resulting model is solved exactly so that no additional accuracy is lost in solution. The use of such models can be quite compelling in appropriate circumstances [1], such as preliminary design, when the concept may be evolving rapidly, or when it is necessary to check solutions developed elsewhere. Moreover, the following review of related work serves to highlight the growing popularity of such techniques.

Approximate methods have recently been developed that can deal with the vibration of asymmetric three-dimensional structures, in which the translational and torsional modes of vibration are coupled. Kuang and Ng [2,3] considered the problem of doubly asymmetric, proportional structures in which the motion is dominated by shear walls. For the analysis, the structure is replaced by an equivalent uniform cantilever whose deformation is coupled in flexure and warping torsion. The same authors extended this concept to the case of wall-frame structures by allowing for bending and shear. In this case however, the wall and frame systems are independently proportional, but result in a non-proportional structural form [4]. In a recent publication, they have extended their work to tall building structures comprising frames, walls, structural cores and coupled walls [5]. As in their previous work, they have replaced the structure with a uniform cantilever, derived the governing differential equation for free vibration and then solved the corresponding eigenvalue problem using a generalised method based on the Galerkin technique. Wall-frame structures have also been addressed by Wang et al. [6], who used an equivalent eccentricity technique that is appropriate for non-proportional structures. However, the analysis is limited to finding the first two coupled natural frequencies of uniform structures with singly asymmetric plan form.

Hand methods have also received considerable attention and are particularly suitable for check calculations. In recent papers by Zalka [7,8], such a method is presented which can deal with the three-dimensional frequency analysis of buildings braced by frameworks, coupled shear-walls and cores.

In a relatively recent publication, Potzta and Kollar [9] replaced the original structure by an equivalent sandwich beam that can

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model both slender and wide structures consisting of frames, trusses and coupled shear walls. In a subsequent paper, an alternative approach is adopted in which the natural frequencies of the replacement beam are solved approximately. This, together with other simplifying assumptions, leads to simple formulae for determining the required natural frequencies [10]. Reference [10] also includes a useful tabulated summary of related work by the following authors [7,11–18].

The most recent contribution has been made by Rafezy et al. [19] who presented a simple, accurate model for the calculation of natural frequencies of asymmetric, three dimensional frame structures whose properties may vary through the height of the structure in a stepwise fashion at one or more storey levels. Their stiffness formulation enables the structure to be modelled as a stepped shear-torsion cantilever which yields the lower natural frequencies for medium to tall structures surprisingly accurately.

The methods developed in the references above offer solutions of varying accuracy, depending on the assumptions employed. Surprisingly, apart from the latter paper, none of them allows for step changes of properties along the height of the structure, despite the fact that this is almost inevitably the case in practical building structures of reasonable height. This study therefore seeks to extend the concept of the paper by Rafezy et al. [19] to wall-core structures.

2. Problem statement

The class of building structure considered herein comprises two sets of orthogonal shear walls that are additionally stiffened by cores whose principal axes run parallel to the same orthogonal directions. Since walls and cores deflect predominantly in a flexural configuration, it is assumed that they obey Bernoulli–Euler bending theory that allows for bending deformation but not shearing deformation. In the case of an asymmetric arrangement of walls and cores, torsional effects are produced and may become significant or even critical in tall buildings. In addition, walls and cores in buildings do not warp freely as they are restrained against warping at foundation level, thus the effect of warping should be taken into account in addition to St. Venant torsion. The warping-restrained torsion is often referred to as Vlasov's torsion and can lead to longitudinal stresses in the walls and cores that are sometimes greater than longitudinal stresses due to overall bending of the structure. In this study both the St. Venant and warping rigidity of the cores are taken into account, but the St. Venant torsional component of the walls, which is dependent on circular shear flows within individual wall elements, is small in comparison and can therefore be ignored.

The underlying approach adopted with the model is to dissect the original building structure into segments, by cutting through the structure horizontally at those storey levels corresponding to changes in storey properties. Thus the storeys contained within a segment between any two adjacent cut planes are identical. A typical segment is then considered in isolation. Initially, one of the two orthogonal plan directions is selected. If there is a primary wall running in this direction it is replaced by an equivalent flexural beam. If there is a core that has a component of flexure in this direction, it is replaced by an equivalent flexure–torsion beam located on the core's original shear centre. In each case, the substitute members have uniformly distributed mass and stiffness. The flexural beam only allows for bending deformation, while the flexure–torsion beam allows for bending deformation, St. Venant and warping torsion. In turn, each additional wall or core that contributes to the structural action in the current direction is replaced by its own substitute beam and the effect of all these beams is summed to model the effect of the original structure. This leads directly to the differential equation governing the motion

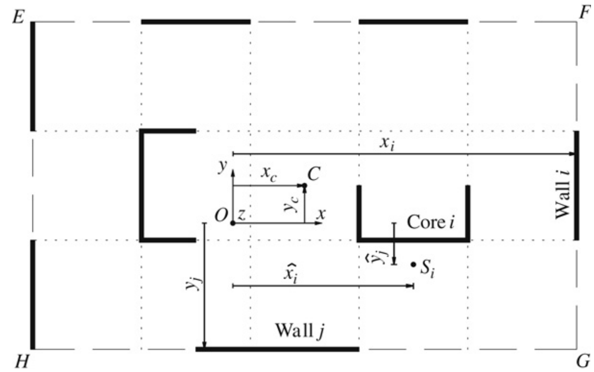

Fig. 1. Typical floor plan of an asymmetric three-dimensional wall-core structure. O and C denote the locations of the rigidity and mass centres, respectively. The floor system $EFGH$ is considered to be rigid in its plane.

of the segment in the chosen direction. The same procedure is then adopted for all the cores and those walls that contribute to the structural action in the orthogonal direction. Once both equations are available it requires little effort to write down the substitute expressions for the coupled torsional motion. The three equations thus formed are subsequently solved exactly and posed in dynamic stiffness form. The resulting coupled flexure–torsion beam element can then be used to reconstitute the original structure by assembling the dynamic stiffness matrices for the individual segments in the usual manner.

It is clear from the element formulation that the final model has a transcendental dependence upon the frequency parameter. The required natural frequencies are therefore determined by solving the model using an exact technique, based on the Wittrick–Williams algorithm, that can be arrested after achieving any desired accuracy and which also ensures that no natural frequencies can be missed.

3. Theory

Consider the hypothetical layout of a typical floor plan of the asymmetric, three-dimensional wall-core structure shown in Fig. 1. The walls run in two orthogonal directions and the cores' principal axes are parallel to these directions. It is assumed that the rigidity centre of the structure, O, at each floor level lies on a vertical line that runs through the height of the structure.

It is now assumed that the origin of the co-ordinate system is located at the rigidity centre, O, with the x and y co-ordinates running parallel to the walls. The z-axis then runs vertically from the base of the building and coincides with the rigidity axis. Point $C(x_c, y_c)$ denotes the centre of mass at a typical floor level. It is assumed that the floor system is rigid in its plane and that the centre of mass at each level lies on a vertical line, the mass axis, that runs through the height of the structure. When the rigidity and mass axes of a structure do not coincide, the lateral and torsional motion of the building will always be coupled in one or more planes.

During vibration, the displacement of the mass centre at any time t in the x–y plane can be determined as the result of a pure translation followed by a pure rotation about the rigidity centre, see Fig. 2. During the translation phase the rigidity centre moves to O' and the mass centre C moves to C', displacements in each case of $u(z, t)$ and $v(z, t)$ in the x and y directions, respectively. During rotation, the mass centre moves additionally from C' to C'', an angular rotation of $\varphi(z, t)$ about O'. The resulting translations, (u_c, v_c) of the mass centre in the x and y directions, respectively, are

$$u_c(z, t) = u(z, t) - y_c\varphi(z, t) \quad \text{and} \tag{1a}$$

$$v_c(z, t) = v(z, t) + x_c\varphi(z, t). \tag{1b}$$