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An approach based on the catenary equation to deal with static analysis of three dimensional cable structures

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ABSTRACT

In this paper a novel method to solve three dimensional cable structures based on the catenary equation is proposed. The method is a generalization of a previous engineering application to compute the initial equilibrium of railway overheads. The major contributions of this paper are: the extension of the previous engineering application to simulate arbitrary three dimensional cable structures; cable elasticity is incorporated into the formulation; and due to the fact that the method relies on the analytical catenary equations, high numerical efficiency is exhibited. In order to show the validity of the method, comparisons with several well reported cable structure problems are presented. The agreement between the proposed method and published results is excellent.

1. Introduction

Due to their efficiency and aesthetics, cable structures became quite popular from the 1950s onwards. From the mid 60s to the end of the 70s a significant number of articles dealing with cable structures were published, see for instance [1–7] among others. Nowadays, cable structures are widely used in many applications as, for example, power transmission lines, railway overheads, cable transportation systems, cable roof structures etc.

Cable structures pose well known challenging problems, and the modelling of such structures has always been a subject of research and innovation. Cable members are light, very flexible and do not experience bending and compression stiffness. Therefore, cable structures exhibit a high non-linear behaviour. Another important problem of cable structures is the determination of the initial equilibrium configuration. That is, the computation of the stressed reference configuration which is an inverse structural problem. Reference [8], is one of the pioneering works dealing with the classification of the methods to solve initial equilibrium problems. Local behaviour of particular types of cables is another quite difficult problem that modelling of cable structures should deal with. Helically wound cables present interwire friction which influences axial stiffness [9]. Cables can show hockling or kinking phenomena as a result of torsional stability of single and double rope systems, [10]. For instance, the validity domain assessment of the mechanical behaviour of simple straight strands which are layers of helical wires wound around a central straight wire core has appeared in [11]. This paper focuses on macro-scale modelling of complex cable structures, that is, the initial equilibrium configuration computation and the cable structure response to external load equilibrium under general loading. Therefore, the modelling of the local behaviour of wire cables is beyond the scope of the paper.

Broadly speaking, the methods used to model cable structures can be classified into two main groups. Following the nomenclature proposed in [8] these approaches are called: the non-linear displacement method and the force density method.

The method of non-linear displacement is based on an iterative process that modifies step by step the geometry from one configuration to another fulfilling the equilibrium equations. Argyris's pioneering work, [12], applies this method to the design of the cable roof of the Olympic Stadium of Munich, replacing real cables with truss elements. The dynamic relaxation method with kinetic damping is used in [13] to determine the initial equilibrium configuration and analyse prestressed nets and membranes. Based on the method of non-linear displacements, some authors, [14] and [15], modelled the cable as a series of straight linear trusses developing specific formulations to improve the method performance. Trying to improve these formulations,

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Karoumi [16] developed a curved finite element, Jayaraman and Knudson [17] formulated a two node finite element based on the exact equation of the elastic catenary and, more recently, Andreu et al. in [18] implemented a deformable catenary element into a finite element method. The tangent stiffness of a cable using the catenary equation is provided by [19]. Finally, it can be said that most of the current non-linear displacement methods are based on the finite element method. An alternative approach to the finite element method is presented in [20] where an engineering application based on the catenary equation is used to analyse the initial equilibrium and stiffness computation of railway overheads.

The force density method was initially introduced by Scheck, [21]. The non-linear nature of the problem is considered by means of the projections of the forces at every node and their non-linear dependency with the nodal coordinates, that are the unknowns of the problem. However, other authors tried to obtain an equivalent linear problem taking into account certain assumptions about the final solution. Among them, Siev and Eidelmann suggest in [1] imposing perpendicularity condition on the horizontal projection of the equilibrium configuration of a cable net. This restriction allows the initial equilibrium to be calculated simply by means of solving an equivalent system of linear equations. However, this assumption only made the method applicable to a reduced range of problems. Indeed, this method was revisited and applied in [22] to solve the initial equilibrium problem of structures composed of mixed cables and rigid elements under compression loads. The force density method was enhanced by Haber and Abel in [8]. Instead of straight beam elements they added curved and shell elements to the force density formulation. The density force method was also combined with least squares optimization techniques to generate the cutting patterns of the membranes that compose a shell structure in [23]. This method is used in [24] to design the shape of deployable membrane reflectors within aerospace applications.

This article deals with the static behavior of three dimensional cable structures. The method proposed in this paper is based on the exact solution of the catenary. The use of the analytic expression of the catenary to solve or formulate complex problems has been used previously by other authors, among others see [25,26,19,18,20,27]. One of the most important differences between the aforementioned models and the herein proposed one is that while previous models use either catenary tension or node positions that are unknown, the herein proposed method uses all possible unknowns in a cable structure problem, that is, the catenary variables such as tension, length and the catenary parameter as well as node positions. In this way, both the initial equilibrium configuration and the cable structure response to external loads can be calculated using the same strategy. The method has been implemented in a general purpose computer code toolbox called CALESCA. Moreover, from this perspective, the proposed formulation should be regarded as a mix between a non-linear displacement method and a force density method.

The remainder of the paper is organised as follows. Section 2 describes the numerical foundations of the method. Section 3 provides the application of the method to several well documented cable structures. The comparison between the method proposed and the published results are also presented. Finally, in Section 4, the main conclusions of this study are summarised.

2. Mathematical formulation

In what follows the mathematical formulation of the method proposed in this paper is presented. First, the global formulation of the catenary equations into a three dimensional reference system is shown. Next, the global equilibrium of a cable structure is defined.

2.1. Global formulation of a single three dimensional cable

As is well-known the catenary is the equilibrium shape of a cable that hangs between fixed points under its own weight. For a comprehensive review of cable mechanics see [28] and for the local catenary formulation followed here see [29]. Let us consider the single three dimensional cable shown in Fig. 1. Using the local reference system $\left\langle \tilde{O}\xi_1\xi_2\xi_3 \right\rangle$, the catenary equation can be written as:

$$\xi_2^i = c\,\cosh\left(\frac{\xi_1^i}{c}\right) \tag{1}$$

where $c = t_1^i/w$ is the catenary shape parameter, t_1^i the horizontal component of the cable tension (direction ξ_1), and w is the weight per unit of length of the cable. The length of the cable between the lowest point of the catenary, l, and a general point, i, is denoted by s^{li} and is defined as

$$s^{li} = c\,\sinh\left(\frac{\xi_1^i}{c}\right). \tag{2}$$

Finally, the tension at point i, t^i, can be expressed by

$$t^i = c\,w\,\cosh\left(\frac{\xi_1^i}{c}\right). \tag{3}$$

The three previous equations, i.e. (1)–(3), summarise the behaviour of a cable under its own weight into a local reference system. Now, the positioning of the single cable should be referred to as a three dimensional frame. The following notation will be used: subscript letters refer to directions and superscript letters refer to a catenary point. Let us consider a general point, i, of the catenary represented in Fig. 1. The spatial position of an arbitrary point i of the catenary, $\boldsymbol{X}^i$, is described by the coordinates with respect to a global cartesian reference system $\langle OX_1X_2X_3\rangle$. This position vector can be expressed in the form

$$\boldsymbol{X}^i = \boldsymbol{X}^{\tilde{O}} + \boldsymbol{R}\cdot\boldsymbol{\xi}^i \tag{4}$$

where $\boldsymbol{\xi}^i \equiv \left(\xi_1^i, \xi_2^i, 0\right)^t$ t is the transpose, and the rotation matrix $\boldsymbol{R}$ is defined by

$$\boldsymbol{R} = \begin{pmatrix} \sin\varphi^{ab} & 0 & -\cos\varphi^{ab} \\ 0 & 1 & 0 \\ \cos\varphi^{ab} & 0 & \sin\varphi^{ab} \end{pmatrix} \tag{5}$$

with $\varphi^{ab} = \operatorname{atan}\left(\left(X_1^b - X_1^a\right)/\left(X_3^b - X_3^a\right)\right)$. The rotation matrix $\boldsymbol{R}$ transforms the vectors from the local system, $\left\langle \tilde{O}\xi_1\xi_2\xi_3 \right\rangle$, to the global system, $\langle OX_1X_2X_3\rangle$. Considering Eqs. (4) and (1) it is possible to express the vertical coordinate of the point i as

$$\xi_2^i = X_2^i - X_2^{\tilde{O}} = c\,\cosh\lambda^i \tag{6}$$

where $\lambda^i = \xi_1^i/c$. Applying Eq. (6) at the two end nodes, a and b, of the catenary arch, see Fig. 1, the following relationship can be found:

$$\xi_2^b - \xi_2^a = X_2^b - X_2^a = c\,\left(\cosh\lambda^b - \cosh\lambda^a\right) \tag{7}$$

where λ^a and λ^b should be written as functions of the unknowns of the problem, that is $\lambda^a(X^a, X^b, c) = \alpha^{ab} - c\,\operatorname{asinh}\left(\beta^{ab}\right)$ and $\lambda^b(X^a, X^b, c) = \alpha^{ab} + c\,\operatorname{asinh}\left(\beta^{ab}\right)$ where

$$\alpha^{ab} = \frac{1}{2}d^{ab} \tag{8}$$