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Nonlinear elastic analysis of composite beams curved in-plan

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ABSTRACT

A novel 3D elastic total Lagrangian formulation is developed for the numerical analysis of steel–concrete composite beams which are curved in-plan. Geometric nonlinearities are considered in the derivation of the strain expressions, and the partial interaction at the interface in the tangential direction as well as in the radial direction due to flexible shear connectors is incorporated in the unique proposed formulation, which is derived from considerations of fundamental engineering mechanics. Examples are presented to illustrate the effects of initial curvature, geometric nonlinearity and partial interaction on the behaviour of composite curved beams, which are compared with those based on more sophisticated but computationally less efficient ABAQUS shell element models and experiments reported in the literature. The results demonstrate that the developed formulation is accurate and effective in capturing the behaviour of composite beams curved in-plan, providing a highly efficient finite element.

1. Introduction

Composite steel and concrete beams which are curved in-plan are used very widely in highway bridges. In deference to curved steel bridges (e.g. [1]), comparatively few studies have been reported on curved composite beams, and in particular on their numerical modelling. Colville [2], Thevendran et al. [3,4] and Shanmugam et al. [5], conducted experiments on steel–concrete composite curved beams to investigate the ultimate load behaviour, while Giussani and Mola [6] recently developed an analytical formulation for elastic composite beams curved in-plan by assuming full interaction between the steel girder and the concrete deck. Chang and White [7] discussed the modelling considerations for composite curved steel bridges and illustrated the effects of cross-sectional distortions.

Studies (e.g. [8,9]) have shown that it is also important to consider geometric nonlinearity in order to accurately predict the response of curved beams, even under service loads. This is because the secondary bending about the minor principal axis and torsion actions may develop and become increasingly profound with an increase of the twist rotations. Bradford et al. [10] showed that during unpropped construction of curved composite bridges, effects of geometric nonlinearity may cause early yielding of the I-girder since I-beam girder acts as a separate individual curved beam when concrete is wet. Thus, time effects are critical for the construction of composite beams. Topkaya et al. [11] conducted experimental and numerical studies to establish the behaviour of composite curved bridges during construction. Liew et al. [12] also showed the effects of geometric nonlinearity on the inelastic behaviour of curved I-beams and proposed simplified equations to evaluate the ultimate strength. The behaviour of curved beams has also been studied by Hall [13], Zureick et al. [14] and Gimena et al. [15–17].

On the other hand, the behaviour of composite beams is significantly influenced by the flexibility of the shear connection. A beam theory that considers the partial interaction in the longitudinal direction of straight beams was presented by Newmark et al. [18]. Adequate modelling of composite curved beams with flexible shear connectors, however, needs to account for the partial interaction in the radial direction as well as in the tangential direction, because in curved beams radial deflections occur even for vertical loading. Accurate beam models for composite beams curved in-plan accounting for partial interaction and incorporating the important coupling of bending and torsion actions and deflections do not appear to have been reported in the open literature. Beam models always have the advantage of easy structural modelling and easy interpretation of the output results. The objective of this paper is therefore to present a 3D geometrically nonlinear beam finite element that considers the partial interaction in the tangential as well as in the radial directions in the analysis of composite beams curved in-plan. Examples are considered to illustrate the effects of initial curvature, geometric nonlinearity and partial interaction on the behaviour of composite curved beams. The proposed finite element formulation is validated by comparing the numerical

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results obtained from the formulation with more sophisticated yet complex ABAQUS shell element models and with available experimental results reported in the literature. The beam finite element is shown to provide a very efficient technique for modelling curved composite beams.

2. Kinematic relations

2.1. Basic assumptions

Fig. 1 shows a composite beam curved in-plan, for which the following assumptions are made:

- The steel girder is a doubly symmetric I-beam that is curved in plan;
- The deck has a rectangular cross-section and has the same initial curvature as the girder in the undeformed configuration;
- Both cross-sections remain rigid throughout the deformation, i.e. no distortion occurs;
- There is no uplifting between the girder and the deck;
- Radius of curvature is constant along the beam;
- The shear connection between the girder and the deck is flexible both in the tangential and radial directions;
- Rotations and deflections are large, but the strains are small;
- According to the fact that plane sections remain normal to the deformed axis, shear strains on the cross-sections are induced by uniform torsion only.

It should be noted that in highway bridges composite curved beams with multi-girder cross-sections are commonly used. For such beams the distortion effects may be significant (e.g. Chang and White [7]) thus the rigid cross-section assumption adopted herein may have to be relaxed when the effects of cross-sectional distortion are significant.

2.2. Position vector

Two coordinate systems are adopted. The fixed (spatial) *oxys* coordinate system has the *s* axis oriented along the axial direction of the curved composite beam (in the un-deformed configuration) while axes *ox* and *oy* are in the plane of the cross-section. The three components of the triad $\mathbf{p}(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_s)$ form an orthogonal basis of the system *oxys* defined along the tangent direction of the axes *ox*, *oy* and *os*, respectively, as shown in Fig. 1. The position vector of a point *P* in the undeformed configuration can be written as

$$\mathbf{a}_0 = \mathbf{r}_0 + x\mathbf{p}_x + y\mathbf{p}_y, \tag{1}$$

where $\mathbf{r}_0$ (Fig. 1) is the position vector of the origin o of the undeformed beam based on an arbitrary but fixed origin O in space. The material (body attached) $o_1x_1y_1s_1$ coordinate system alters with the deformation of the structure so that coordinates of a point on the beam after deformation will be identical to those before deformation. The material coordinate system coincides with the spatial coordinate system only in the undeformed configuration. The total deformations of origin o_1 are considered to result from translations due to the displacements u, v, and w along the tangent direction of the axes *ox*, *oy* and *os*, respectively, and a finite rotation of the total cross-section through an angle ϕ about the axis *os*. Hence, the position vector $\mathbf{r}$ (Fig. 1) of the origin o_1 is

$$\mathbf{r} = \mathbf{r}_0 + u\mathbf{p}_x + v\mathbf{p}_y + w\mathbf{p}_s. \tag{2}$$

The position vector of the point *P* at the deformed configuration can be written as

$$\mathbf{a} = \mathbf{r} + x\mathbf{q}_x + y\mathbf{q}_y - \omega(x, y)\left[\kappa_s(s) + \Omega_\kappa(s)\right]\mathbf{q}_s + \Omega_s(s)\mathbf{q}_s + \Omega_x(s)\mathbf{q}_x, \tag{3}$$

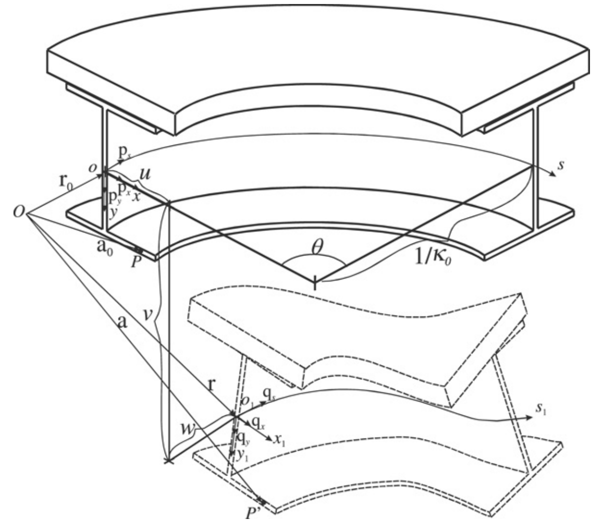

Fig. 1. Coordinate systems, position vectors and displacements.

where the triad $\mathbf{q}(\mathbf{q}_x, \mathbf{q}_y, \mathbf{q}_s)$ is positioned along the tangent direction of the deformed axes o_1x_1, o_1y_1 and o_1s_1, respectively and $\omega(x, y)$ is the normalised section warping displacement function (e.g. [19]). In Eq. (3), κ_s is the twist ratio about the undeformed axis s and the deformations Ω_x and Ω_s are due to slip displacements in the radial and tangential directions in the horizontal plane, respectively. The function Ω_κ results from the slip between the top flange of the steel girder and the deck during the warping action of the cross-section. Herein, the functions Ω_x, Ω_s and Ω_κ are assumed positive for the girder and negative for the deck. The total slip displacements between the girder and the deck u_{sp} and w_{sp} in the x (radial) and s (tangential) directions, respectively can thus be obtained by subtracting the position vector of the deck from that of the girder, i.e.

$$u_{sp}(s) = 2\Omega_x(s) \tag{4}$$

and

$$w_{sp}(x_i, y_i, s) = 2\Omega_s(s) - 2\omega(x_i, y_i)\Omega_\kappa(s), \tag{5}$$

in which x_i and y_i denote the coordinates of a point at the interface.

2.3. Finite rotations and strains

The finite rotation tensor $\mathbf{R}$ determines the orientation of the triad $\mathbf{q}$ with respect to the triad $\mathbf{p}$, i.e. $\mathbf{q} = \mathbf{R}\mathbf{p}$. The components of $\mathbf{R}$ in terms of displacements u, v, w and the angle of twist ϕ are given in Appendix A. In the deformed configuration, the curvatures κ_x, κ_y and the twist ratio κ_s about the deformed axes o_1x_1, o_1y_1 and o_1s_1, respectively can be obtained from Serret–Frenet formulae [20] as

$$\begin{Bmatrix} \mathrm{d}\mathbf{q}_x/\mathrm{d}s \\ \mathrm{d}\mathbf{q}_y/\mathrm{d}s \\ \mathrm{d}\mathbf{q}_s/\mathrm{d}s \end{Bmatrix} = (1+\varepsilon)\begin{bmatrix} 0 & \kappa_s & -\kappa_y \\ -\kappa_s & 0 & \kappa_x \\ \kappa_y & -\kappa_x & 0 \end{bmatrix}\begin{Bmatrix} \mathbf{q}_x \\ \mathbf{q}_y \\ \mathbf{q}_s \end{Bmatrix}. \tag{6}$$

The components of κ_x, κ_y and κ_s are also given in terms of u, v, w, and ϕ in Appendix A. Several stress and strain measures for the geometric nonlinear analysis of structures are established in the literature. In order to develop a total Lagrangian finite element formulation, the Green–Lagrange strains are adopted herein. In line with the rigid cross-section assumption, the normal strains in the x and y directions are assumed to be zero and the non-zero normal strain on the cross-sectional surface can be calculated from

$$\varepsilon_{ss} = \frac{1}{2}\left(\frac{\mathrm{d}\mathbf{a}}{\mathrm{d}s}\frac{\mathrm{d}\mathbf{a}}{\mathrm{d}s} - \frac{\mathrm{d}\mathbf{a}_0}{\mathrm{d}s}\frac{\mathrm{d}\mathbf{a}_0}{\mathrm{d}s}\right). \tag{7}$$