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Reliability-based optimal design of linear dynamical systems under stochastic stationary excitation and model uncertainty

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ABSTRACT

The reliability-based design under stochastic stationary excitation of linear dynamical systems with higher-dimensional output is discussed in this paper. An analytical approximation is initially presented for the calculation of this reliability for applications for which the model parameters are treated as known. This approach is based on a computationally efficient approximation to the conditional out-crossing rate for higher-dimensional vectors. The extension to cases where the model parameters are treated as unknown and are characterized by probability models is then addressed. This requires the evaluation of a multidimensional probability integral over the uncertain model parameter space and a methodology based on a Taylor series expansion around the local maxima of the integrand, called design points, is considered for this purpose. A novel approach is developed for addressing cases with multiple design points. The estimation of this probability integral by Monte Carlo simulation with importance sampling is also considered. Implementation details for applications to reliability-based design problems are extensively discussed. In particular, the effect of the errors introduced by the various, numerical and asymptotic, approximations is addressed and methods for reducing their relative or absolute importance are presented. Also practical guidelines are provided for improvement of the computational efficiency for using the analytical reliability approximation within the algorithm that searches for the optimal system design configuration.

1. Introduction

Calculation of system reliability is one of the most difficult problems in stochastic analysis of dynamical systems. This task is often referred to as solving the first-excursion (or first-passage) problem and it is defined as the determination of the probability that within some given time duration, the output trajectory of a system out-crosses the boundary of a safe region that defines acceptable performance. The system model parameters, based on the available knowledge, may either be treated as certain or include some level of uncertainty. The latter can be quantified by assigning appropriate probability models to them [1]. The interest in this area focused originally on analytical approximations for dynamical systems with known model characteristics and low-dimensional output. First-excursion problems for scalar processes have therefore received a lot of attention (e.g. [2,3]), while the vector process counterpart has received much less attention. Theoretical discussions [4], including computation of lower and upper bounds [5,6], have been made, but practical results for the multidimensional case were initially presented only for applications with independent processes and very small dimensions [7,8], presumably because the computational capabilities required for the associated calculations were not available at the time that the interest was focused on this topic. Over the last decade or so, stochastic simulation methods have dominated the research interest in this area [9–12]. These methods offer the ability to estimate system reliability with a predefined level of accuracy, can be easily extended to additionally address uncertainties in the system model properties, apart from the uncertainty stemming from the stochastic excitation, and can efficiently handle complex description for the dynamical system (for example, nonlinearities in the response) and for the stochastic excitation (for example, nonstationary characteristics).

The recent applications of reliability-based concepts for the design of engineering systems under stochastic excitation [13–15] have created an incentive for researchers to revisit the problem of analytical evaluation of the reliability of dynamical systems. Such design applications require a large number of evaluations of the system reliability with a small – or at least consistent – estimation error, when performing the search to identify the optimal system properties. Analytical approximations are frequently an attractive alternative to stochastic simulation for this

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Nomenclature

$\boldsymbol{\theta}$	Vector of model parameters
Θ	Space of possible values for $\boldsymbol{\theta}$
$p(.)$	Probability distribution function
$\boldsymbol{\varphi}$	Vector of design variables
Φ	Admissible space for design variables
$\boldsymbol{\varphi}^*$	Optimal design choice
$\mathbf{x}$	State vector
$\mathbf{w}$	White-noise vector
$\mathbf{A}(\boldsymbol{\varphi};\boldsymbol{\theta}), \mathbf{E}(\boldsymbol{\varphi};\boldsymbol{\theta}), \mathbf{C}(\boldsymbol{\varphi};\boldsymbol{\theta})$	State Space Matrices
$\mathbf{z}$	Output vector
z_i	ith performance variable
β_i	Acceptable response threshold for z_i
$\mathbf{n}_i$	Outward normal vector for $z_i = \beta_i$ plane
$\mathbf{o}_i$	Orthogonal to $\mathbf{n}_i$ component of $\mathbf{z}$
$\mathbf{T}$	Matrix transformation between $\mathbf{o}_i$ and $\mathbf{z}$
D_s	Acceptable performance domain for $\mathbf{z}$
S_D	Boundary of D_s
B_i	Hyperplane pair $\|z_i\| = \beta_i$
Δ_i	Intersection of D_s and B_i
$P(.)$	Probability
$P_F(\boldsymbol{\varphi})$	Failure probability
$P_F(\boldsymbol{\varphi}\|\boldsymbol{\theta})$	Failure probability for specified $\boldsymbol{\theta}$
$E[.]$	Expected value
$n_{\mathbf{z}}^+(\boldsymbol{\varphi}, t; \boldsymbol{\theta})$	Conditional mean out-crossing rate for $\mathbf{z}$
$\nu_{\mathbf{z}}^+(\boldsymbol{\varphi};\boldsymbol{\theta})$	Stationary conditional mean out-crossing rate
$r_{z_i}^+(\boldsymbol{\varphi};\boldsymbol{\theta})$	Unconditional mean out-crossing rate for z_i
$\lambda_{z_i}(\boldsymbol{\varphi};\boldsymbol{\theta})$	Temporal correlation factor for z_i
$P_{z_i}(\boldsymbol{\varphi};\boldsymbol{\theta})$	Spatial correlation weighting factor for z_i with respect to $\mathbf{z}$
$\mathbf{P}(\boldsymbol{\varphi};\boldsymbol{\theta})$	State covariance matrix
$\mathbf{K}_{\mathbf{zz}}(\boldsymbol{\varphi};\boldsymbol{\theta})$	Output covariance matrix
$\sigma_{z_i}^2$	Stationary variance for z_i
$\sigma_{\dot{z}_i}^2$	Stationary variance for $\dot{z}_i$
I	Probability integral
$k(\boldsymbol{\varphi},\boldsymbol{\theta})$	Response function involved in I
$s(\boldsymbol{\theta};\boldsymbol{\varphi})$	Log of the integrand of I
$\mathbf{H}_s(\boldsymbol{\theta};\boldsymbol{\varphi})$	Hessian matrix for $s(\boldsymbol{\theta};\boldsymbol{\varphi})$
$\boldsymbol{\theta}_j^*$	jth design point
$g_j(\boldsymbol{\theta})$	Gaussian approximation around $\boldsymbol{\theta}_j^*$
I_j	Probability integral of scaled Gaussian approximation around $\boldsymbol{\theta}_j^*$
RI_j	Additional contribution to I from Gaussian approximation around $\boldsymbol{\theta}_j^*$
r_j	Reduction factor for I_j
$\boldsymbol{\theta}_i$	Sample for model parameters
$q(\boldsymbol{\theta})$	Importance sampling density

purpose and a resurgence of interest on the analytical reliability calculation has been demonstrated [16–18]. For example, for linear systems under stationary excitation analytical approximations can facilitate a computationally efficient estimation of the system reliability [18], which is highly appropriate for the aforementioned type of design problems.

This paper focuses (i) on an analytical approximation for the stationary reliability of certain and uncertain linear dynamic systems with higher-dimensional output, and primarily (ii) on computational aspects for the application of this approximation to reliability-based design. The latter constitutes and the main contribution of this work. The system reliability is directly adopted as the objective function. Although reliability-based design optimization problems are frequently formulated by adopting deterministic objective functions, for example the structural cost, and using the system reliability as a constraint [19,20], cases that involve the system reliability directly as the objective functions are also common; for example such problems are encountered in the field of structural control [15,21]. Note that the analytical approximation presented here could be also used for design applications that involve the stationary reliability as a constraint. The discussions here, though, focus on computational and theoretical considerations when this reliability is selected as the performance objective to be optimized.

Initially the approximation by Taflanidis and Beck [18] for the first-passage problem is reviewed. This approximation estimates the system reliability using the conditional out-crossing rate over the boundary of the region that defines acceptable performance for the system output. Numerical issues related to the evaluation of this rate and practical considerations pertaining to reliability-based design are discussed. The extension to systems with uncertain model parameters is then presented. This requires evaluation of multidimensional probability integrals and an existing asymptotic expansion is suggested for this task. This expansion involves solving for the local maxima of the integrand, called design points, and accurately calculating the Hessian matrix at these locations. A novel method is developed here to address applications that involve multiple design points. An alternative approach, based on Monte Carlo integration, is also discussed for evaluation of the multidimensional probability integral. The implications on robust stochastic design are extensively discussed, when the above approaches are used for evaluation of system reliability. In particular, the effect of the errors introduced by the numerical and asymptotic approximations is addressed and methods for reducing their relative or absolute importance are presented. Also relevant practical guidelines are provided for improvement of the computational efficiency within the algorithm that searches for the optimal system design.

2. Problem formulation

Consider a linear system subject to stochastic excitation that is modeled as filtered Gaussian white noise. The system includes model parameters $\boldsymbol{\theta} \in \Theta \subset \Re^{n_\theta}$, and some adjustable parameters which define its design, referred to herein as design variables, $\boldsymbol{\varphi} \in \Phi \subset \Re^{n_\varphi}$, where Φ denotes the bounded admissible design space. The state space form of the system is

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(\boldsymbol{\varphi};\boldsymbol{\theta})\mathbf{x}(t) + \mathbf{E}(\boldsymbol{\varphi};\boldsymbol{\theta})\mathbf{w}(t)\\ \mathbf{z}(t) &= \mathbf{C}(\boldsymbol{\varphi};\boldsymbol{\theta})\mathbf{x}(t)\end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \Re^{n_x}$ is the state vector; $\mathbf{z}(t) \in \Re^{n_z}$ is the vector of performance variables (output of the system); $\mathbf{w}(t) \in \Re^{n_w}$ is a vector of zero-mean Gaussian white-noise disturbances, appropriately normalized, so that the spectral intensity matrix equals to the identity matrix $\mathbf{I}$; and $\mathbf{A}(\boldsymbol{\varphi};\boldsymbol{\theta})$, $\mathbf{E}(\boldsymbol{\varphi};\boldsymbol{\theta})$, $\mathbf{C}(\boldsymbol{\varphi};\boldsymbol{\theta})$ are matrices that are a function of the design variables $\boldsymbol{\varphi}$ and depend on the model parameters $\boldsymbol{\theta}$. The state vector $\mathbf{x}(t)$ is an augmentation of the original structural system states, together with ancillary states related to the stochastic input model and any other dynamics pertaining to the system. Thus, the formulation in (1) takes into account the spectral characteristics of the stochastic excitation by appropriate augmentation of the state equation.

In the space of the performance variables $\mathbf{z}(t)$, we consider an hyper-rectangular domain $D_s \subset \Re^{n_z}$ that defines acceptable performance:

$$D_s = \left\{\mathbf{z}(t) \in \Re^{n_z} : |z_i(t)| < \beta_i,\ \forall\, i = 1, \ldots, n_z\right\}. \tag{2}$$

An example for a three-dimensional space is shown in Fig. 1. Region D_s is bounded by the hyperplane pairs $B_i : |z_i| = \beta_i, i = 1, \ldots, n_z$ and by appropriate definition of the output vector $\mathbf{z}(t)$ can represent any symmetric limit state function. The extension to