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Structural damage identification by using wavelet entropy

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ABSTRACT

The wavelet transform is combined with Shannon entropy to detect structural damage from measured vibration signals in this paper. Damage features such as wavelet entropy, relative wavelet entropy and wavelet-time entropy are defined and investigated to detect and locate damage. The damage identification method is examined by a numerically simulated case and two laboratory test cases. It is demonstrated that wavelet-time entropy is a sensitive damage feature in detecting the abnormality in measured successive vibration signals, while relative wavelet entropy is a good damage feature to detect damage occurrence and damage location through the vibration signals measured from the intact (reference) and damaged structures. In addition, the relative wavelet entropy method is flexible in choosing the reference signal simultaneously measured from any undamaged location of the target structure. This advantage is particularly interesting in detecting the damage of existing structures because data from the intact (undamaged) structure is not required.

1. Introduction

During the service of structures such as large-scale frames, long-span bridges and high-rise buildings, local damage of their key locations may continually accumulate, and finally results in structural failure. One damage assessment classification commonly defines four levels of damage identification [1,2]: (1) the presence of damage; (2) the location of damage; (3) quantification of the severity of damage; and (4) prediction of the remaining serviceability of the structure. Basically, damage detection techniques can be classified into either local or global methods. Most currently used techniques, such as visual, acoustic, magnetic field, eddy current etc. are effective. They are "local" inspection approaches in nature. Structural damage detection through changes in dynamic characteristics, on the other hand, provides a "global" way to evaluate the structural state. The problem is always formulated as: given the changes in the structural dynamic characteristics before and after the damage, to predict the location and severity of damage.

One core issue of global vibration-based damage identification methods is to seek some damage features that are sensitive to structural damage [3,4]. The damage features that have been demonstrated with various degrees of success include natural frequencies, mode shapes, mode shape curvatures, modal flexibility, modal strain energy, etc. Doebling et al. [1] summarized the comprehensive historic development of damage assessment methodologies based on these features as well as pointing out their applicability and limitations. Most vibration-based structural damage identification methods require the modal properties that are extracted from measured signals through system identification techniques. It is realistic to extract structural damage features directly from the measured vibration signals.

Structural damage is typically a local phenomenon. Fourier analysis transforms the vibration signal from a time-based or space-based domain to a frequency-based one. So, it is sometimes impossible to determine when or where a particular event takes place by using Fourier transforms (FT). To improve this deficiency, the short-time Fourier transform (STFT) that was proposed by Gabor [5] can be used. This windowing technique analyzes only a small section of the signal at a time. The STFT maps a signal into a 2-D function of time or space and frequency. The transformation has the disadvantage that the information about time or space and frequency can be obtained with limited precision that is determined by the size of the window. A higher resolution in both time and frequency domain cannot be achieved simultaneously since once the window size is fixed, it is the same for all frequencies.

The wavelet transform (WT) is a new technique to analyze the signals. Wavelet functions are composed of a family of basis functions that are capable of describing a signal in localized time (or space) and frequency (or scale) domain [6]. The main advantage

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gained by using wavelets is the ability to perform local analysis of a signal, i.e., zooming on any interval of time or space. Signal-based damage detection techniques that involve wavelet analysis take advantage of this to be capable of revealing some hidden aspects of measured signals. Many investigators presented the applications of wavelet transform [7–12] or wavelet packet transform [13–15] to detect cracks or damage in structures.

Damage will result in disorder of a structural response signal. A natural approach to quantify the degree of order of a complex signal is to look into its spectral entropy, as defined from the Fourier power spectrum [16]. Spectral entropy is akin to a representation of concentration of the Fourier power spectrum of a signal. An ordered activity, like a sinusoidal signal, is manifested as a narrow peak in the frequency domain. This concentration of the frequency spectrum in one single peak corresponds to a low entropy value. On the other extreme, a disordered activity (e.g. the one generated by pure noise or by a deterministic chaotic system) will have a wide band response in the frequency domain, this being reflected in higher entropies. However, the applicability of this method to short lasting and non-stationary data segments has restrictions.

A time evolving entropy can be defined from a time–frequency representation of the signal as provided by the wavelet transform [17–20]. The orthogonal discrete wavelet transform makes no assumptions about record stationarity and the only input needed is the time series. In this case, the time evolution of frequency patterns can be followed with an optimal time–frequency resolution. Therefore, while entropy based on the wavelet transform reflects the degree of order/disorder of the signal, it can provide additional information about the underlying dynamical processes associated with the signal [21].

The objective of this paper is intended to combine wavelet transform with information entropy to fully use the advantages of both techniques. A wavelet entropy-based method is proposed for damage detection of structures. Damage features such as wavelet entropy (WE), relative wavelet entropy (RWE) and wavelet-time entropy (WTE) are defined and investigated to detect and locate damage. Wavelet entropy is used to characterize the degree of order/disorder associated with a multi-frequency signal response. Relative wavelet energy is defined to provide information about the relative energy associated with different frequency bands presented in the structural response segments, while wavelet-time entropy is used to describe time evolution of wavelet entropy. Both numerically simulated and tested beams in the laboratory with different damage scenarios are used as case studies to validate the applicability of the proposed damage identification procedure. Results have demonstrated that the wavelet entropy-based index is a good candidate of damage features that is sensitive to structural local damage.

2. Theoretical background

2.1. Wavelet transform

Wavelet analysis is a signal processing method, which relies on the introduction of an appropriate basis and a characterization of the signal by the distribution of amplitude in the basis. If the wavelet is required to form a proper orthogonal basis, it has the advantage that an arbitrary function can be uniquely decomposed and the decomposition can be inverted [6,13]. The wavelet is a smooth and quickly vanishing oscillating function with good localization in both frequency and time. A wavelet family $\psi_{a,b}(t)$ is the set of elementary functions generated by dilations and translations of a unique admissible mother wavelet $\psi(t)$:

$$\psi_{a,b}(t) = |a|^{-1/2}\,\psi\left(\frac{t-b}{a}\right) \tag{1}$$

where $a, b \in R$, $a \neq 0$, are the scale and translation parameters, respectively, and t is time. As the scale parameter a increases, the wavelet becomes wider. Thus, one has a unique analytic pattern and its replications at different scales and with variable time localization.

The continuous wavelet transform of a signal $S(t) \in L^2(R)$ (the space of real square summable functions) is defined as the correlation between the function $S(t)$ with the family wavelet $\psi_{a,b}(t)$ for each a and b:

$$(W_\psi S)(a,b) = |a|^{-1/2}\int_{-\infty}^{\infty} S(t)\psi^*\left(\frac{t-b}{a}\right)\mathrm{d}t = \langle S, \psi_{a,b}\rangle . \tag{2}$$

For a special election of the mother wavelet function $\psi(t)$ and for the discrete set of parameters, $a_j = 2^{-j}$ and $b_{j,k} = 2^{-j}k$ with $j, k \in Z$ (the set of integers) the family

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k) \quad j, k \in Z \tag{3}$$

constitutes an orthonormal basis of the Hilbert space $L^2(R)$ consisting of finite-energy signals. The correlated decimated discrete wavelet transform provides a non-redundant representation of the signal and its values constitute the coefficients in a wavelet series. These wavelet coefficients provide full information in a simple way and a direct estimation of local energies at different scales. Moreover, the information can be organized in a hierarchical scheme of nested subspaces called multi-resolution analysis in $L^2(R)$. In the present work, orthogonal cubic spline functions are employed as mother wavelets. Among several alternatives, cubic spline functions are in a suitable proportion with smoothness and numerical advantages and they have become a recommended tool for representing natural signals.

In the following, the signal is assumed to be given by the sampled values $S = \{s_0(n), n = 1, \ldots, M\}$, corresponding to a uniform time grid with sampling time t_s. For simplicity the sampling rate is taken as $t_s = 1$. If the decomposition is carried out over all resolutions levels, $N = \log_2 M$, the wavelet expansion will be:

$$S(t) = \sum_{j=-N}^{-1}\sum_k C_j(k)\psi_{j,k}(t) = \sum_{j=-N}^{-1}\gamma_j(t) \tag{4}$$

where wavelet coefficients $C_j(k)$ can be interpreted as local residual errors between successive signal approximations at scales j and $j+1$, while $\gamma_j(t)$ is the residual signal at scale j. It contains information of the signal $S(t)$ corresponding to frequencies $2^{j-1}\omega_s \le |\omega| \le 2^j\omega_s$.

2.2. Wavelet energy

Since the family $\{\psi_{j,k}(t)\}$ is an orthonormal basis for L^2 (R), the concept of energy is linked with the usual notions derived from Fourier theory. Then, wavelet coefficients are given by $C_j(k) = \langle S, \psi_{j,k}\rangle$, and the energy of a signal at each scale $j = -1, \ldots, -N$, will be

$$E_j = \|\gamma_j\|^2 = \sum_k |C_j(k)|^2 . \tag{5}$$

The energy at each sampled time k will be

$$E(k) = \sum_{j=-N}^{-1} |C_j(k)|^2 . \tag{6}$$

In consequence, the total energy can be obtained by

$$E_{tol} = \|S\|^2 = \sum_{j<0}\sum_k |C_j(k)|^2 = \sum_{j<0} E_j . \tag{7}$$