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Large amplitude flexural vibration analysis of tapered plates with edges elastically restrained against rotation using DQM

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ABSTRACT

The large amplitude free vibration analysis of tapered rectangular thin plates with edges elastically restrained against rotation is investigated using a differential quadrature method (DQM). The governing equations are based on the thin plate theory using Green's strain in conjunction with von Karman assumption. In order to better recognize the nonlinearity effects, the in-plane immovable conditions are assumed along the edges of the plate. The boundary conditions are exactly implemented at the boundary grid points and are conveniently built into the equations of motion using a DQ methodology recently developed by the authors to solve fourth-order governing differential equations. The harmonic balance method is used to transform the resulting differential equations from temporal to frequency domain. Consequently, a direct iterative method is used to solve the nonlinear eigenvalue system of equations. The convergence of the method is verified and the solution accuracy is demonstrated by comparing the results with those for limiting cases, i.e., the free vibration of tapered plates under classical boundary conditions. Furthermore, the effects of different parameters on the nonlinear to linear natural frequency ratio of plates with linearly or bi-linearly varying thickness and with edges elastically restrained against rotation are studied and the results are compared with those of DQ solution based on the first-order shear deformation plate theory.

1. Introduction

Linear and nonlinear free vibrations of rectangular thin plates with uniform thickness have been extensively studied by many investigators. For tapered thin plates however, only linear free vibrations have been conducted, see for example Refs. [1–9]. Due to much practical use of tapered plates and panels in structures under large deformation, the subject is of interest in many practical applications.

It is well known in experimental studies on plate vibration that the exact simulation of the so-called simply supported edges is impossible, since the edges will always experience some amount of resistance to rotation. Therefore, such constraints have promoted new modeling procedures with improved solutions to account for better agreement with those of practical data. In comparison, with nonlinear free vibration analyses of plates subjected to classical boundary conditions, only limited studies are available for plates with elastically restrained edges [10–12] all limited to plates with uniform thickness.

The Galerkin and the finite element methods as the two common methods have been used to carry out a discretization in the spatial domain in studying the nonlinear free vibration analyses of plates. The differential quadrature method (DQM) as an alternative numerical technique has been used in such structural analyses [13–19]. In this respect, Feng and Bert [13] studied the nonlinear free vibration of isotropic Euler–Bernoulli beams using the conventional DQM. Using a spline based DQM, Guo and Zhong [14] investigated the same problems considered by Feng and Bert [13]. In another work, they considered the nonlinear free vibration analysis of isotropic Timoshenko beams [15]. Liew and his co-workers [16–23] have introduced and solved various types of analyses for two- and three-dimensional plate structures by DQM as extended applications of this powerful algorithm in structural mechanics analysis. Li and Cheng [24] studied the effects of large deformation on fundamental natural frequency of rectangular orthotropic plates using DQM. More recently, Malekzadeh investigated the nonlinear free vibration analyses of skew composite plates [25,26] and tapered Mindlin plates [27]. In all these studies, uniform thickness beams and plates with simply supported and clamped edges have been considered.

In this paper, the applicability of a DQ approach is investigated for solving large amplitude free vibration of tapered thin plates with elastically restrained against rotation edges. Conventional DQM cannot handle thin walled structural problems in a

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Nomenclature

a	plate dimension in x-direction
A	extensional stiffness of plate at an arbitrary point (x, y)
$A_{ij}^{x(r)}$	DQ weighting coefficient of the rth-order derivative in x-direction
A_{ij}^{x}, A_{ij}^{y}	DQ weighting coefficients of the first-order derivative in x- and y-directions, respectively
b	plate dimension in y-direction
B_{ij}^{x}, B_{ij}^{y}	DQ weighting coefficients of the second-order derivative in x- and y-directions, respectively
D	bending stiffness of plate at an arbitrary point
D_o	bending stiffness of plate at the corner $x = y = 0$
E	Young's modulus of plate
h_o	thickness of plate at the corner $x = y = 0$
k_{ir}	elastic restraint coefficient along the edge 'i' of plate $(i = 1, \ldots, 4)$
K_{ir}	non-dimensional coefficient of rotational stiffness along the edge 'i' of plate [$= k_{ir}D_o/b$ along x-edges and $k_{ir}D_o/a$ along y-edges]
M_{xx}, M_{yy}, M_{xy}	bending moment about y- and x-axes and twisting moment, respectively
n_x, n_y	x- and y-components of unit normal vector to an arbitrary edge of plate
N_x, N_y	number of grid points in x- and y-directions
N_{xx}, N_{yy}	in-plane normal force resultant in x- and y-directions
$(N_{xx})_{ij}, (N_{yy})_{ij}$	discretized in-plane normal force resultant in x- and y-directions
N_{xy}	in-plane shear force resultant
$(N_{xy})_{ij}$	discretized in-plane shear force resultant
u, v, w	displacement components in x- y- and transverse-directions of a point on mid-plane of plate, respectively
$\{u\}_d, \{v\}_d$	in-plane displacement vectors at domain grid points
$\{w\}_d$	transverse displacement vector at domain grid points
x, y, z	the Cartesian coordinate variables
W_c	center deflection of plate
ν	Poisson's ratio of plate
ε_o	convergence tolerance
ρ	plate density

straightforward manner, as one of the drawbacks of DQM. This is because their equations of motion include fourth-order differential equations and consequently multiple boundary conditions of the field variable do exist at boundary grid points [28,29]. To overcome this drawback, a recently developed DQM by Karami and Malekzadeh [30,31] was employed for the imposition of such boundary conditions. It was shown that the boundary conditions can be built into the discretized form of the equations of motion without using the usual matrix partitioning for eliminating the boundary degrees of freedom or reformulations of DQ weighting coefficients. Also, in spite of some conventional DQ approaches [32], the boundary conditions are exactly implemented at boundary grid points. In this paper, the convergence of this DQM will be demonstrated and its accuracy will be checked by comparing the results for nonlinear free vibration of plates with limiting cases of boundary conditions and also for the linear free vibration problems available in the literature. As no solution is available for nonlinear free vibration of tapered plates, the results are compared with those of the first shear deformation theory based (FSDT) DQM method. This is because the FSDT has the second-order partial differential equations and hence has no problem with boundary condition implementation [18,19]. This work therefore establishes the applicability of the aforementioned DQ methodology for a relatively complicated engineering analysis problem, i.e. nonlinear free vibration analysis of tapered thin plates.

2. Governing equations

A plate of length a, width b and thickness h is considered (see Fig. 1). Based on the classical thin plate theory (TPT), the constitutive relations for the resultant in-plane forces and moments can be related to the displacement components (u, v, w) at an arbitrary point on the mid-plane of the plate as,

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = A \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{(1-\nu)}{2} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix},$$
$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = -D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu) \end{bmatrix} \begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ \dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{1}$$

where $A = Eh/(1-\nu^2)$ and $D = Eh^3/12(1-\nu^2)$.

The in-plane equations of motion for a tapered rectangular plate in terms of displacement components in the rectangular Cartesian coordinate system in x- and y-directions can be written as follows, respectively,

$$\begin{aligned} & h\frac{\partial^2 u}{\partial x^2} + \frac{\partial h}{\partial x}\frac{\partial u}{\partial x} + \frac{(1-\nu)}{2}h\frac{\partial^2 u}{\partial y^2} + \frac{(1-\nu)}{2}\frac{\partial h}{\partial y}\frac{\partial u}{\partial y} + \frac{(1+\nu)}{2}h\frac{\partial^2 v}{\partial x \partial y} \\ & + \nu\frac{\partial h}{\partial x}\frac{\partial v}{\partial y} + \frac{(1-\nu)}{2}\frac{\partial h}{\partial y}\frac{\partial v}{\partial x} + \frac{1}{2}\frac{\partial h}{\partial x}\left(\frac{\partial w}{\partial x}\right)^2 + h\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} \\ & + \frac{\nu}{2}\frac{\partial h}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{(1+\nu)}{2}h\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{(1-\nu)}{2}h\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2} \\ & + \frac{(1-\nu)}{2}\frac{\partial h}{\partial y}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = \frac{(1-\nu^2)\rho h}{E}\frac{\partial^2 u}{\partial t^2} \end{aligned} \tag{2}$$

$$\begin{aligned} & \left(\frac{1+\nu}{2}\right)h\frac{\partial^2 u}{\partial x \partial y} + \nu\frac{\partial h}{\partial y}\frac{\partial u}{\partial x} + \left(\frac{1-\nu}{2}\right)\frac{\partial h}{\partial x}\frac{\partial u}{\partial y} + h\frac{\partial^2 v}{\partial y^2} + \frac{\partial h}{\partial y}\frac{\partial v}{\partial y} \\ & + \left(\frac{1-\nu}{2}\right)h\frac{\partial^2 v}{\partial x^2} + \left(\frac{1-\nu}{2}\right)\frac{\partial h}{\partial x}\frac{\partial v}{\partial x} + \frac{1}{2}\frac{\partial h}{\partial y}\left(\frac{\partial w}{\partial y}\right)^2 + h\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2} \\ & + \frac{\nu}{2}\frac{\partial h}{\partial y}\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{1+\nu}{2}\right)h\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y} + \left(\frac{1-\nu}{2}\right)h\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial y} \\ & + \left(\frac{1-\nu}{2}\right)\frac{\partial h}{\partial x}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = \frac{(1-\nu^2)\rho h}{E}\frac{\partial^2 v}{\partial t^2}. \end{aligned} \tag{3}$$

For tapered thin plate, the transverse equation of motion (in the z-direction) and the rotational equations of motion about the x- and y-axes can be summarized as

$$\begin{aligned} & -D\nabla^4 w - 2\frac{\partial D}{\partial x}\frac{\partial(\nabla^2 w)}{\partial x} - 2\frac{\partial D}{\partial y}\frac{\partial(\nabla^2 w)}{\partial y} - \left(\frac{\partial^2 D}{\partial x^2} + \nu\frac{\partial^2 D}{\partial y^2}\right)\frac{\partial^2 w}{\partial x^2} \\ & - \left(\frac{\partial^2 D}{\partial y^2} + \nu\frac{\partial^2 D}{\partial x^2}\right)\frac{\partial^2 w}{\partial y^2} - 2(1-\nu)\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} \\ & + N_{xx}\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_{yy}\frac{\partial^2 w}{\partial y^2} = \rho h\frac{\partial^2 w}{\partial t^2}. \end{aligned} \tag{4}$$