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# Inelastic finite element analysis of composite beams on the basis of the plastic hinge approach

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ABSTRACT

This paper emphasizes material nonlinear effects on composite beams with recourse to the plastic hinge method. Numerous combinations of steel and concrete sections form arbitrary composite sections. Secondly, the material properties of composite beams vary remarkably across its section from ductile steel to brittle concrete. Thirdly, concrete is weak in tension, so composite section changes are dependent on load distribution. To this end, the plastic zone approach is convenient for inelastic analysis of composite sections that can evaluate member resistance, including material nonlinearities, by routine numerical integration with respect to every fiber across the composite section. As a result, many researchers usually adopt the plastic zone approach for numerical inelastic analyses of composite structures. On the other hand, the plastic hinge method describes nonlinear material behaviour of an overall composite section integrally. Consequently, proper section properties for use in plastic hinge spring stiffness are required to represent the material behaviour across the arbitrary whole composite section. In view of numerical efficiency and convergence, the plastic hinge method is superior to the plastic zone method. Therefore, based on the plastic hinge approach, how to incorporate the material nonlinearities of the arbitrary composite section into the plastic hinge stiffness formulation becomes a prime objective of the present paper. The partial shear connection in this paper is by virtue of the effective flexural rigidity as AISC 1993 [American Institute of Steel Construction (AISC). Load and resistance factor design specifications. 2nd ed., Chicago; 1993]. Nonlinear behaviour of different kinds of composite beam is investigated in this paper, including two simply supported composite beams, a cantilever and a two span continuous composite beam.



## 1. Introduction

It is common in most buildings that steel beams support concrete floor slabs, which is designed to act compositely with the steel beam by using shear connections at the interface. A partial degree of continuity between concrete slabs and steel beams is inevitably presented due to the selection of the strength, stiffness, and spacing of connectors. Partial shear connection makes the composite design of the structures become more economical. Further, a composite structure with partial shear connections behaves with greater ductility, so it is helpful to carry out a plastic design and seismic design. Composite beams with partial shear connection are therefore popular structural forms in most composite buildings.

When the distribution of bending moment changes along a composite beam, the degree of continuity between the steel beam and concrete slab also varies along the member according to moment distribution. Partial interaction between different components also causes nonlinear effects on the composite beam. One of the earliest governing differential equations of partial shear connection for one dimensional elastic composite beam was developed by Newmark el al. [2], which relies on force equilibrium at the interface between steel and concrete. After three decades, Arizumi et al. [3] developed finite element analysis of composite beams with incomplete shear connections on the basis of a displacement-based stiffness approach. However, the conventional displacement-based element is unable to accurately solve the partial shear interaction effect. A study by Neuenhofer and Filippou [4] indicated that the use of force-based element in the numerical formulation can improve the accuracy of numerical results of the partial shear connection effect to a significant extent.

To overcome this weakness of displacement-based stiffness approach, some researchers, including Daniels and Crisinel [5] and Salari et al. [6], also separately developed a nonlinear finite element analysis based on a forced-based flexibility formulation, which in principle relies on interpolation of internal forces within the element so that the equilibrium condition at the interface is strictly enforced along members for geometrically

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linear problems. Difficulties arise, however, in the selection of force interpolation functions that strictly satisfy the equilibrium condition between beam displacements and internal forces. In other words, the force-based flexibility formulation alone, just like the displacement-based stiffness formulation, lacks strong interaction between displacements and internal forces where no deformation interpolation functions relate the deformations along the element to nodal displacement. Therefore, in order to solve the behaviour of partial shear connections of composite beams, Ayoub and Filippou [7] proposed a consistent mixed formulation to exploit the advantages of displacement and force formulations. Since the displacement and force variables are independent, the member resisting forces, however, cannot be directly computed from deformation. And, thus, the mixed formulation is much complicated and intricate. Alemdar and White [8] summarized displacement-based, flexibility and mixed formulation of composite element. Another approach to solve the partial degree of continuity between steel beams and concrete slabs is AISC [1], which is represented by effective flexural rigidity empirically such that nonlinear analysis for composite beams becomes simpler and more versatile.

Besides partial shear interaction, another significant nonlinear effect on composite members is material behaviour, which varies along the composite beam dependent of loading distribution. In the sagging moment region, the steel beam may usually yield due to tension and the concrete slab crushes by compression. In the hogging moment region, the concrete in tension is cracked and reinforcement inside the concrete slab may yield normally due to traction. Consequently, nonlinear material behaviour of beams is different along the member length when subjected to either sagging or hogging moments.

For inelastic analysis of composite structures, many researchers, including El-Tawil and Deierlein [9] and Spacone et al. [10,11], adopted the plastic zone approach because of its advantage of solving material yielding not only along composite members, but also across the diverse composite section routinely by virtue of numerical integration. Nevertheless, the solution from the plastic zone method requires much computational effort and time. On the contrary, the plastic hinge method obtains a reasonably accurate solution for composite structures with less computational effort and is an efficient process. Based on the plastic hinge approach, Liew et al. [12] studied inelastic behaviour of steel frames with composite beams. The plastic hinge formulation in that study, which is developed and modified from Li et al. [13] and Attalla et al. [14], can model strain-hardening and the spread of the yield effect on a steel beam composite supporting concrete slab. The plastic hinge approach can advantageously obtain a solution of material nonlinearity with an efficient convergent rate.

In regard to the material nonlinear effect on composite sections, the plastic zone approach is suitable for modelling the material behaviour of every fiber across the section conveniently and routinely. However, the plastic zone method requires greater computational effort with an inefficient convergent rate. On the other hand, the plastic hinge approach relies on proper section properties to establish its spring stiffness, which can integrally represent nonlinear material behaviour of an entire composite section in line with numerical integration in the plastic zone method. The plastic hinge method can ensure efficient convergence without sacrifice of accuracy. Hence routine numerical integration (plastic zone method) with respect to every fiber across the section is conceptually replaced by the proper section properties (plastic hinge method) of arbitrary whole composite sections. How to evaluate proper section properties for plastic hinge spring stiffness becomes crucial for determining different material behaviour on arbitrary composite sections as a whole. Therefore, this paper presents a plastic hinge approach by using proper section properties in order that the proposed plastic hinge spring stiffness can simulate the material nonlinearities of the whole composite section, which includes gradual yielding, full plasticity and strain-hardening effect on composite beams in general. Cracking of concrete sections is accommodated by using a diminishing concrete section. It is worth noting that section properties of composite beams should be evaluated dependent of the loading distribution, including sagging and hogging moments. To obtain a reasonably accurate solution of composite action with numerical efficiency, the present nonlinear analysis of composite structure simulates the partial shear interaction by making use of the effective flexural rigidity according to AISC [1].

## 2. Basic assumptions of the formulation

The following assumptions are made for the stiffness formulation of the present approach,

(1) The beam is prismatic and slender, and the Euler–Bernoulli hypothesis is valid,
(2) Warping deformation, shear deformation and twisting effect are also neglected, and no lateral–torsional buckling of steel beam,
(3) Loads are assumed to be independent of the load path and incrementally increase proportionally,
(4) Nodal load response is only included in the present formulation,
(5) Capacity of concrete under tension is negligible,
(6) Partial shear connection is uniformly distributed along the member, disregarding various modes of slip along the interface of the composite beam,
(7) Shear load to slip relationship of shear stud is linear,
(8) No separation between steel beam and concrete slab is allowed such that they have a same curvature,
(9) Element is elastic and all material nonlinearities are allowed for in plastic hinge spring.

## 3. Basic stiffness formulation of cubic element

The basic stiffness formulation of the beam element in the present analysis is by virtue of a cubic displacement function to relate the displacements of composite beam elements with internal force distribution. Linear curvature of the element is therefore obtained, so a reasonable number of elements should be used in the nonlinear analysis to represent a composite beam member, when the bending moment distribution is nonlinear along the member.

The elastic force-displacement relationship is derived from the total potential energy of the composite element. The total potential energy for nonlinear analysis of composite beam is written in Eq. (1).

$$\Pi = U - V$$

$$\begin{aligned}\prod = {} & \frac{EA}{2}\int_L\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2\mathrm{d}x + \frac{P}{2}\int_L\left(\frac{\mathrm{d}v}{\mathrm{d}x}\right)^2\mathrm{d}x + \frac{P}{2}\int_L\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2\mathrm{d}x \\ & + \frac{EI_z}{2}\int_L\left(\frac{\mathrm{d}^2v}{\mathrm{d}x^2}\right)^2\mathrm{d}x + \frac{EI_y}{2}\int_L\left(\frac{\mathrm{d}^2w}{\mathrm{d}x^2}\right)^2\mathrm{d}x + \frac{GJ}{2}\int_L\left(\frac{\mathrm{d}\gamma}{\mathrm{d}x}\right)^2\mathrm{d}x \\ & - \{d_k\}^{\mathrm{T}}\{f_k\}\end{aligned} \tag{1}$$

in which $u, v$ and $w$ are the axial deformation and lateral displacements in the direction in $y$-axis and $z$-axis, respectively. $\gamma$ is independent twist rotation about the $x$-axis. $EA$, $EI$ and $GJ$ are the axial rigidity, flexural rigidity about corresponding axes and torsional rigidity, respectively. $P$ is the axial member load. And $\{d_k\}$ and $\{f_k\}$ are the column vectors of the displacement and