Engineering Structures 30 (2008) 2923–2934

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Simple solutions for the flexural-torsional buckling of laterally restrained I-beams

A. Khelil *, B. Larue

Nancy Université, LEMTA CNRS UMR 7563, IUT Nancy Brabois, Civil Engineering Department, Le Montet CS 90137 - 54601, Villers-Les-Nancy cédex, France

ARTICLE INFO

Article history:
Received 20 September 2007
Received in revised form
19 March 2008
Accepted 26 March 2008
Available online 12 May 2008

Keywords:
Flexural-torsional buckling
Beam
Lateral restraints

ABSTRACT

This paper describes a method for analysing the lateral buckling of beams continuously restrained at one flange. The deformed configuration is governed by a system of differential equations solved by the Galerkin method. The matrices of rigidities, restraints and geometry are specified as well as the problem with the eigenvalues, with a view to predicting the buckling loads and the corresponding buckling shapes. The effects of moment distribution and continuous restraints on the elastic flexural-torsional buckling of beams are also studied. The results show that the restraint of the tensioned flange can have a weak influence on the critical buckling moment.

1. Introduction

Flexural-torsional buckling is an important limit state that must be considered in structural steel design. This type of instability phenomena occurs when a structural member undergoes significant out-of-plane bending and twisting. The failure occurs suddenly in members with a much greater in-plane bending stiffness than torsional or lateral bending stiffness.

The problem of lateral torsional buckling of steel beams has been studied extensively by many authors, including Timoshenko and Gere [1–8]. In these investigations, the critical load is determined either by integrating the governing differential equations or by making use of an energy principle. However, these investigations are mainly focused on the buckling of an isolated beam without considering secondary structural members (such as sheeting, steel deck). A beam is often connected to other elements which participate in the buckling actions and significantly influence the structure's buckling resistance. This configuration is encountered very frequently in practice, as in the case of purlins supporting a steel deck cover, and also in that of cross bars of a portal frame with purlins.

Continuous lateral restraints are usually considered as being uniform along the length of a beam, and are often used to approximate the actions of restraining elements connected to the beam at closely spaced intervals, as in the case of roof sheeting. When continuous restraints are assumed to be rigid, they impose a longitudinal axis about which the beam cross-sections rotate during buckling (Larue & Khelil [9,10]). Among these investigations, mention can be made of the work done by Trahair, and Hancok [11], Bradford [12–14], Nethercot [15] and Trahair [16].

More recent research on the theory of flexural-torsional buckling has been presented by Tong and Zhang [17,18] with their investigations of a new theory to clarify the inconsistencies of existing theories of the flexural-torsional buckling of thin-walled members. In the general case of restrained beams, exact solutions for buckling loads cannot be obtained and a numerical method must be used to obtain an approximate solution.

This paper presents a simple numerical theory designed to resolve the governing differential equations of instability for this problem. The analysis presented uses the Galerkin method to determine the critical load. The matrices of rigidities, restraints and geometry are specified as well as the problem with the eigenvalues in order to predict the buckling loads and the corresponding buckling shapes.

The calculation details are specified voluntarily so that readers can summarise and adapt the study to each particular case. Several cases of distributed forces are treated as linearly varying loads and parabolic varying bending moments.

This method cannot be applied to welded I-section beams with slender webs, as lateral-distortional buckling may occur. A computationally efficient method is presented by Bradford [19] for studying this phenomenon. In addition, certain problems within this field of lateral restrained beam–column stability, are investigated by Horne & Ajmani [20].

* Corresponding author. Tel.: +33 383 68 25 36; fax: +33 383 68 25 32.
E-mail address: Abdel.khelil@iutnb.uhp-nancy.fr (A. Khelil).

0141-0296/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2008.03.017

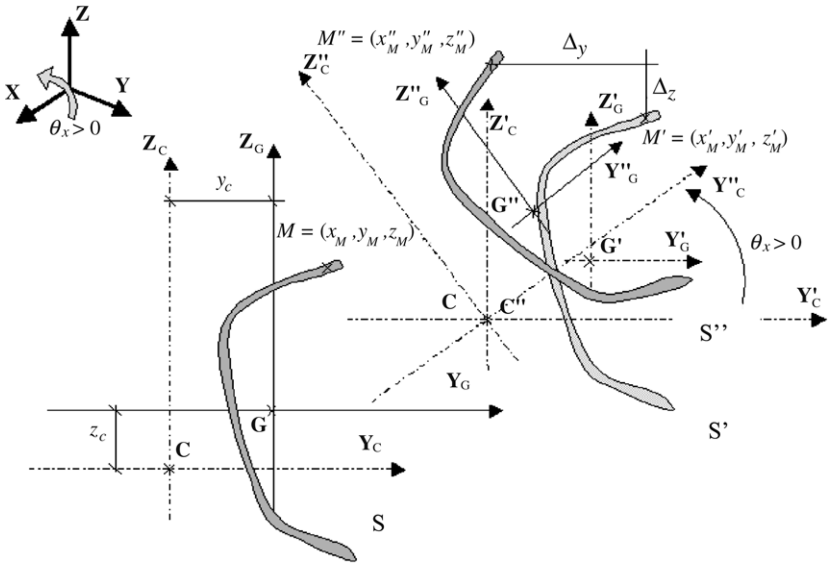

Fig. 1. Displacement and twist rotation for an asymmetric section.

2. Kinematics

At the critical state there is a tendency for the compression flange to bend sideways and for the remainder of the cross-section, which is stable, to restrain it. The net effect is that the section rotates and moves laterally, as shown in the Fig. 1. Here, the total displacement of the cross-section is composed of 2 translations and one rotation [21].

A straight thin-walled beam with an arbitrary cross-section member is referenced by a Cartesian-coordinate system (x, y, z), where the x-axis is parallel to the longitudinal axis of the beam and y and z are the principal axes of the cross-section.

After deformation, section (S') results from (S) by two translations $\vec{v}(x)$ and $\vec{w}(x)$.

$v(x)$ is the displacement of point C along axis $(C, \vec{y})$ in coordinate system $R_C\,(C, \vec{y}_C, \vec{z}_C)$.

$w(x)$ is the displacement of point C along axis $(C, \vec{z})$ in coordinate system $R_C\,(C, \vec{y}_C, \vec{z}_C)$.

Cross-section (S'') results from (S') by the rotation of $\theta_x(x)$ of centre O'.

(y_C, z_C) are the shear centre coordinates.

The displacement of point $M\,(x_M, y_M, z_M)_G$ is then given by

$$\overrightarrow{MM}'' = \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\text{Translation}} \begin{bmatrix} \vec{X}_G & \vec{Y}_G & \vec{Z}_G \end{bmatrix} + \underbrace{\begin{bmatrix} y_M \sin(\theta_z) + z_M \sin(\theta_y) \\ (y_M - y_C)\,[\cos(\theta_x) - 1] - (z_M - z_C)\sin(\theta_x) \\ (y_M - y_C)\sin(\theta_x) + (z_M - z_C)\,[\cos(\theta_x) - 1] \end{bmatrix}}_{\text{Rotation}} \times \begin{bmatrix} \vec{X}_G & \vec{Y}_G & \vec{Z}_G \end{bmatrix}. \quad (1)$$

For moderate rotation, we can then write:

$$\begin{cases} \sin(\theta_x) \approx \theta_x \\ \cos(\theta_x) - 1 \approx -\dfrac{1}{2}\theta_x^2. \end{cases}$$

According to these hypotheses, the displacement field is assumed to take the following form

$$\overrightarrow{MM}'' = \begin{bmatrix} u - y_M\theta_z + z_M\theta_y \\ v - (z_M - z_C)\,\theta_x - \dfrac{1}{2}(y_M - y_C)\,\theta_x^2 \\ w + (y_M - y_C)\,\theta_x - \dfrac{1}{2}(z_M - z_C)\,\theta_x^2 \end{bmatrix} \times \begin{bmatrix} \vec{X}_G & \vec{Y}_G & \vec{Z}_G \end{bmatrix}. \quad (2)$$

3. Strain field

Assumptions

- The cross-section contour is rigid.
- The shear deformation is disregarded.
- The twisting angle about the shear centre is reasonably small, thus $\theta_x^2 \approx 0$.
- The warping function of the section is constant along the beam.
- The displacement about the x-axis due to warping (defined according to Vlassov's theory) is equal to $\omega_C \frac{\partial\theta_x}{\partial x}$ where ω_C is the section warping function.
- The material is elastic, isotropic and homogeneous.

According to these hypotheses, the displacement field is simplified as:

$$\begin{aligned} \bar{u}(x, y, z) &= u - y\theta_z + z\theta_y + \omega_C \frac{\partial\theta_x}{\partial x} = u - y\frac{\partial v}{\partial x} - z\frac{\partial w}{\partial x} + \omega_C\frac{\partial\theta_x}{\partial x} \\ \bar{v}(x, y, z) &= v - (z - z_c)\,\theta_x \\ \bar{w}(x, y, z) &= w + (y - y_c)\,\theta_x. \end{aligned} \quad (3)$$

The stress–strain field

Components ε_{xx}, ε_{yy} and ε_{zz} of Green's strain tensor are given:

$$\begin{aligned} \varepsilon_{xx} &= \left[\frac{\partial\bar{u}}{\partial x}\right]_L + \frac{1}{2}\left[\left(\frac{\partial\bar{u}}{\partial x}\right)^2 + \left(\frac{\partial\bar{v}}{\partial x}\right)^2 + \left(\frac{\partial\bar{w}}{\partial x}\right)^2\right] \\ \varepsilon_{yy} &= \left[\frac{\partial\bar{v}}{\partial y}\right] + \frac{1}{2}\left[\left(\frac{\partial\bar{u}}{\partial y}\right)^2 + \left(\frac{\partial\bar{v}}{\partial y}\right)^2 + \left(\frac{\partial\bar{w}}{\partial y}\right)^2\right] \\ \varepsilon_{zz} &= \left[\frac{\partial\bar{w}}{\partial z}\right] + \frac{1}{2}\left[\left(\frac{\partial\bar{u}}{\partial z}\right)^2 + \left(\frac{\partial\bar{v}}{\partial z}\right)^2 + \left(\frac{\partial\bar{w}}{\partial z}\right)^2\right]. \end{aligned} \quad (4)$$