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Analytical–numerical study of interfacial stresses in plated beams subjected to pulse loading

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Abstract

In strengthening an existing beam using externally bonded soffit plates, interfacial stresses play an important limiting role on the attainment of full flexural capacity of the composite system. This paper deals with spatial and temporal distributions of shear and normal interfacial stresses in a plated beam subjected to pulse loading. Interfacial stresses are related to displacement coordinates using a set of assumptions and the coupled partial differential equations in the space of these displacement coordinates have been derived by application of dynamic equilibrium, compatibility and constitutive relations. The equations have been solved numerically for the dynamic response. The results for three pulse loading cases have been compared to finite element analyses and reasonable correlation is observed. Compared to dynamic or quasi-static ranges the correlation is weaker when loading is in the impulsive range. Spatial stress distribution suggests that unlike static loading, in the case of pulse loading interfacial stresses are not necessarily concentrated at the edges.

Keywords: Analytical model; Debonding; Interfacial stress; Finite element analysis; UDL

1. Introduction

An existing beam can be strengthened or retrofitted by bonding a steel or fibre-reinforced plastic (FRP) plate to its soffit (Fig. 1). This plate-bonding technique has been used to strengthen or rehabilitate reinforced concrete (RC) and steel beams as well as beams of other materials [1–6]. The advantage of this method is that work can be carried out fairly simply and quickly with negligible disruption and minimum changes to member dimensions. The success of this method has been confirmed experimentally and reported [3,4,6]. Nevertheless, there are limitations in the application of this technique. One of the key issues regarding the effectiveness of this method of strengthening, whether the bonded plates are of steel or FRP materials, is that of debonding. Interfacial shear and normal stresses are developed in a strengthened beam under loading which must pass through the adhesive (resin) layer to be transferred to the plate. The adhesive layer has less robustness than the beam and strengthening plate and thus forms the weak link. Debonding of the soffit plate from the beam deters the full ultimate flexural capacity of the composite system from being achieved. Failure due to debonding under static loads has been addressed by researchers [2,3,6].

Researchers [7–19] have proposed simple analytical expressions to evaluate interfacial shear and normal stresses. All existing solutions are based on two fundamental assumptions which enable relatively simple closed-form solutions to be obtained. The first assumption is that materials of the beam, strengthening plate and resin are all linearly elastic. The second assumption is that stresses are constant across the thickness of the adhesive layer.

Smith and Teng [10] studied the relationship between the existing solutions and proposed a solution based on necessarily the same assumptions. They derived explicit expressions for stresses for three types of loading viz. uniformly distributed load (UDL), arbitrarily placed single-point load and four-point bending. Rasheed and Pervaiz [12] formulated the second-order differential equation of interface shear using the assumptions of beam theory with a shear deformable adhesive layer for fully and partially plated beams and for three types of loading viz. UDL, three-point bending and four-point bending. However,

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Nomenclature

English

a	length of strengthening plate
a_{ij}	generalized coordinates
A_a, A_b	cross-sectional areas of materials a and b
b_c	width of the soffit plate
d_c	thickness of the adhesive layer
e	distance between origin of coordinates in materials a and b
E_a, E_b, E_c	Young's moduli of materials a, b and adhesive
$E_f^{\mathrm{I}}, E_f^{\mathrm{II}}$	fracture energies for modes I and II
F_f^n	force required to cause failure in tension (mode I)
F_f^s	force required to cause failure in shear (mode II)
F	vector of known values/functions
f_a, f_b	shear forces in materials a and b
G_c	shear modulus of the adhesive
I_a, I_b	moment of inertia of materials a and b about their centroids
i	Latin index
j	Latin index
K_n, K_s	normal and shear stiffnesses per unit length of connection
L	length of the beam/differential operator
$\mathscr{L}$	Laplace transform operator
$\mathscr{L}^{-1}$	inverse Laplace transform operator
m_a, m_b	moments in materials a and b
$q(x,t)$	arbitrary line load intensity field
$q(t)$	uniform line load intensity field
r	non-composite length of the beam
t	time
t_a, t_b	axial forces in materials a and b
t_d	pulse load duration
T_f	time to separation in time to failure model
T_n	natural period of the fundamental relevant vibration mode
t_r	rise time to maximum temporal of the pulse load
u_a^*, u_b^*	total displacement of materials a and b in the x-direction
u_a, u_b	displacements of materials a and b in the x-direction at the origin of coordinates
W	weight function vector
w_1	displacement of non-composite length in the z-direction
w_a, w_b	displacements of materials a and b in the z-direction
x	coordinate axis in the longitudinal beam direction
z	coordinate axis in the transverse beam direction
z_a, z_b	distance from the origin of coordinates in materials a and b
Z_{ia}, Z_{ib}	z-coordinate of interface in materials a and b

Greek

α	Greek index
β	Greek index
δ	incremental symbol
$\varepsilon_a, \varepsilon_b$	axial strains in materials a and b
φ	shape function (on spatial distribution)
Ω_1	domain of unstrengthened part
Ω_2	domain of strengthened part
σ_a, σ_b	axial stresses in materials a and b
$\bar{\sigma}_n$	normal force per unit length in adhesive layer
$\bar{\tau}_s$	shear force per unit length in adhesive layer
$\dot{H} = \frac{\partial H}{\partial t}$, $H_{,x} = \frac{\partial H}{\partial x}$	temporal and spatial derivatives

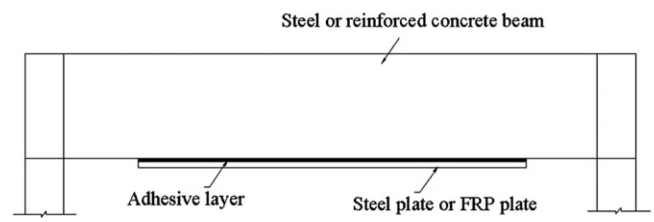

Fig. 1. Schematic of a concrete/steel beam strengthened on the tension part by externally bonded steel/FRP plate.

their work does not include normal stresses; neither does the shear stress satisfy the zero shear stress condition at the end of the adhesive layer. Vilnay [16], Liu and Zhu [17], Taljsten [18] and Malek et al. [19] considered the compatibility of deformations to determine the interfacial stresses. Interfacial shear stresses in the adhesive layer are related to the difference between the longitudinal displacement at the base of the beam and at the top of the soffit plate. The differences among these solutions in determination of interfacial shear stresses are due to the inclusion of different terms in determining these longitudinal displacements. Interfacial normal stresses are related to vertical deformation compatibility between the beam and the bonded plate. Vilnay [16] and Taljsten [18] derived the governing equation in terms of the vertical displacement of the bonded plate. Liu and Zhu [17] and Malek et al. [19] derived the governing equation in terms of interfacial normal stress. However, these equations are interrelated and can be derived from one another with some differences. Roberts and Haji-Kazemi's solution [9] is for UDL only. It is a two- stage solution giving explicit expressions for both interfacial shear and normal stresses at each stage. The superposition of the two stages will lead to the complete result. However, normal stresses at stage one as well as shear stresses at stage two are negligible.

None of the analyses discussed above satisfies the zero shear stress condition at the end of the adhesive layer. For this condition to be satisfied, a higher-order analysis has to be carried out. The first such analysis has been conducted by Rabinovich and Frostig [20]. Nevertheless, this analysis does not provide explicit expressions for interfacial stresses. The correctness of this analysis has also been questioned [15].

Interfacial shear stresses obtained from first-order beam theory and from the analytical methods mentioned above are almost the same except for the region around the free edge of the plate. However, this small region is crucial as it is