Engineering Structures 30 (2008) 3193–3203

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Instability analysis of offshore towers in waves

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A R T I C L E I N F O

Article history:
Received 2 February 2008
Received in revised form
27 April 2008
Accepted 28 April 2008
Available online 9 June 2008

Keywords:
Offshore structures
Buoyant tower
Stability
Waves
Single point mooring
Experiment

A B S T R A C T

This paper investigates the instability of an upright buoyant tower subjected to surface waves. In this regard, Duffing's equation and Mathieu type instability are discussed in the analysis of moored floating structures. In particular, the stability of a special purpose single point mooring (SPM) concept is discussed in detail. A single leg inclined mooring (SLIM) tower was developed as a concept for an articulated structure to moor tankers with a single hawser in shallow water. The tower is not given enough buoyancy to stay upright and assumes an inclined position under equilibrium and thus possesses a circle of equilibrium. Inside this circle, the tower is found unstable. The interaction of this tower with incident waves is analyzed here. It is shown that the frequency domain analysis applicable to upright articulated tower is generally not applicable for SLIM. The amplitude of inline oscillation predicted by this solution is found to be conservative and time domain solution is desired that can retain the nonlinearities. The time domain solution explained the observed SLIM oscillation in waves. In the case of SLIM attached to a tanker, the frequency-domain analysis for a coupled system is shown to correlate the experimental data well and generally no instability is observed.

1. Introduction

Mechanical systems may produce global aperiodic (amplitude modulated quasi-period and chaotic) behavior. The response from such systems due to dynamic loads from fluid flow grows when damping in the system is small. Coupling of various degrees of freedom and the shedding of vortices further complicates the behavior of the systems. For an offshore mooring system, the magnitude of this response due to an aperiodic behavior can be significant.

For the production and storage of offshore oil, a variety of floating and bottom-mounted structures are built offshore. In moderate water depths submerged pipelines are common means of bringing the crude oil from the offshore production platform to the nearby shore for processing. However, sometimes the placement of pipelines on the ocean floor is difficult or unaffordable and alternative techniques are sought in transporting this oil. One of the common means of offloading crude offshore is the placement of a floating structure in the form of a floating buoy or an articulated tower that can be used to moor shuttle tankers [1]. These structures are attached to the ocean floor and are designed to withstand the environment experienced at the site. The shuttle tanker gets attached to these towers/buoys by a single mooring line for offloading of oil. This setup is generally called a single point mooring (SPM) system. When such a system encounters waves, it may undergo instability [8]. The stiffness of the single point line is generally highly nonlinear and the instability primarily occurs because of this high nonlinearity of the mooring line. In high waves the line often becomes slack because of the differential movement between the tower and the tanker. The stiffness characteristics of the line causes system instability under certain environmental conditions. Nonlinear stiffness is responsible for subharmonic response of the tower.

Nonlinear aperiodic response has been reported in several investigations, as reported by Kyozuka et al. [12], Eatock Taylor and Knoop [9], and Sharma et al. [16]. Gottlieb [10] considered a simple dynamic system in the form of a buoyant sphere attached to the ocean floor with a single spring. He discussed the instability behavior of this mechanical system in the vertical plane subjected to planar waves. The parametric resonance of spar platforms was reported by Haslum and Faltinsen [11] in which the relevance of Mathieu instability and associated motion amplification of spar was assessed. The occurrence of parametric resonance in a mono column structure was studied recently by Mattoso [13].

Another example of a floating structure, which often experiences similar instability, is a tanker in its roll motion. The damping of a tanker in roll is generally quite small. In addition, the roll natural period of a tanker falls in the area of frequency band of waves of significant energy. This produces excessive rolling of the tanker in a beam sea. The damping and restoring moments in roll are nonlinear. Roberts [14] studied the stability of the nonlinear equation of motion in roll of a tanker and presented the phase-plane diagram showing instability region.

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0141-0296/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2008.04.034

Several remedial actions have been proposed to improve the seakeeping performance of a tanker in roll and reducing its roll response. Some of these modifications are

- Increase the span and width of bilge keels of the ship,
- Install a free surface roll stabilization flume tank on the deck of the tanker for tuning the roll period,
- Incorporate a "SLO-ROL" system by cross connecting the port and starboard sponsons of the tanker in a Frahm configuration.

The SLO-ROL system provides, in particular, a much-reduced response of the tanker.

Examples of subharmonic motion of a tower in irregular waves and corresponding hawser tension were given by Chakrabarti [6]. This oscillation due to excitation gives rise to the Matheiu type instability. Chantrel and Marol [7] analyzed this type of instability for a single point mooring system. In this case the stiffness term arising from the tower buoyancy and the hawser has a linear-plus-cubic form [6]. The equation of motion reduces to a Matheiu equation, which provides a stable, and an unstable response region whose relative areas directly depend on the amount of overall damping present in the system. Experimental data presented by Chantrel and Marol [7] demonstrated that the response might jump from the stable to an unstable region. From the design point of view of such system, this type of instability may be a major consideration.

2. Theory of SPM

This paper examines the stability problem of an articulated tower having nonlinear elements. For this purpose the instability of the articulated tower with and without the presence of a tanker is investigated. Duffing's equation is discussed in the analysis of the unstable motion of the tower. A single leg inclined mooring (SLIM) tower was investigated as a concept for an articulated structure to moor tankers with a single hawser in shallow water. The tower assumes an inclined position under equilibrium and, thus possesses a circle of equilibrium. Inside the circle, the tower is unstable. The interaction of incident waves with the tower alone and the tower connected to a tanker in a single point moored configuration is analyzed here. In particular, the instability behavior of the tower subjected to waves is studied. It is shown that the equation of motion applicable to upright articulated tower is generally not applicable for SLIM. The amplitude of inline oscillation predicted by this solution is found to be conservative.

A wave tank test of a model of SLIM was performed to demonstrate the instability problem. It is found that the tower takes on a stable motion in small waves. However, when the waves grow larger, the motion appears chaotic. The transverse oscillation is observed to be large in this case. When a tanker was attached to SLIM through a single point mooring, the tower was further inclined, particularly due to a steady current and wave drift load experienced by the tanker. Under this condition, the tower was found to have a stable condition. Based on this study, it was recommended that the concept is not a viable solution for use as an SPM and subsequently was not pursued further.

3. Stability analysis of a moored submerged buoy

Buoys near the sea surface provide similar facilities as an articulated tower. Unlike towers, which are rigidly connected to the ocean floor, floating buoys are attached to the bottom with mooring lines (Fig. 1). Similar to the tower, buoy mooring systems are characterized by a nonlinear restoring force, a nonlinear velocity-dependent damping force and an exciting force. The restoring force includes material discontinuities and geometric nonlinearities associated with large amplitude motion [10]. The external force includes nonlinear effects and periodic components governed by steady (e.g., current) and unsteady (e.g., wave-induced) viscous drag, radiation damping and convective inertia effects [6]. While the response of these nonlinear systems to external and parametric excitation may be periodic, they may also exhibit aperiodic motion. Considering a planar motion of the buoy in the *XY* (*X* horizontal, *Y* vertical) plane, the three degrees of freedom, namely surge, pitch and heave, are coupled. Similar to articulated towers, the system dissipative force includes a coupled quadratic wave-induced damping governed by the relative motion. Thus, the internal forces of the system subjected to an external wave force will include an inertia force including the added mass term, a nonlinear relative velocity drag damping force, and a nonlinear coupled restoring force.

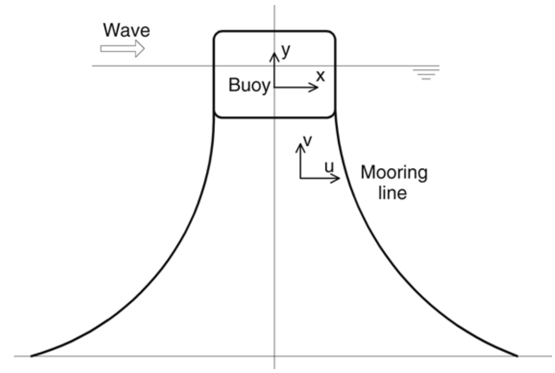

Fig. 1. Set-up of a buoy mooring system.

Let us consider a floating buoy moored to the ocean bottom by soft mooring lines of stiffness k. The system is described in Fig. 1. The system is free to respond in the two-dimensional vertical plane of the wave propagation, x and y. No transverse or lift effect is considered in the analysis. Then the equation of motion is written [10] as three degrees of freedom system. For example, in the propagating direction of wave (x-direction), the equation of motion has the form

$$m\ddot{x} + \rho V C_A \ddot{x} + C\dot{x} + \frac{1}{2}\rho C_D A\sqrt{(\dot{x}-u)^2 + (\dot{y}-v)^2}(\dot{x}-u) + R(x,y)x = \rho V(1+C_A)\left[\dot{u} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x}\right] \quad (1)$$

in which m = mass of the buoy, C_A = hydrodynamic mass, C = linear damping, C_D = hydrodynamic drag coefficient, R = restoring force, V = displaced volume of buoy, u, v = horizontal and vertical velocity components and the dots represent derivative with respect to time. The various terms in the above equation are shown explicitly. The first and second terms are the structure and added mass inertia and C_A is the added mass coefficient (=0.5 for a sphere). The third and fourth terms are the linear and quadratic damping terms. The nonlinear damping term is based on the relative velocity normal to the buoy. The fifth term is the restoring force of the moored system, which is dependent on the position of the buoy in reference to its equilibrium position. The right hand term is the exciting force based on the water particle acceleration and the convective inertia term. Note that the convective inertia term arises from the Navier–Stokes equation and is included when a nonlinear wave theory is applied. The last term may be significant, if the displacement of the buoy is comparable to the submerged weight of the mooring lines. The quantities u and v are the horizontal and vertical water particle velocities by the nonlinear Stokes wave theory. Eq. (1) is normalized by dividing by the buoy mass, $(m + \rho V C_A)$. Then, the equation of motion in the X-direction