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On the prediction of the collapse load of circular concrete columns confined by FRP

M. Fraldi *, L. Nunziante, F. Carannante, A. Prota, G. Manfredi, E. Cosenza

Department of Structural Engineering, University of Napoli "Federico II", Napoli, Italy

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ABSTRACT

The present work is aimed at deriving assessment and design formulae for determining the elastic–plastic response and the ultimate compressive strength of circular concrete columns confined by Fiber Reinforced Polymers (FRP). To achieve this, a constructive method for obtaining closed-form elastic and post-elastic solutions for Functionally Graded Material Cylinders (FGMCs), constituted by an isotropic central core and n arbitrary cylindrically orthotropic hollow phases, is proposed. In the first part of the paper, under the hypotheses of axis-symmetrical boundary conditions, elasticity and perfect bond between the phases, new analytical solutions for self-equilibrated axial forces applied at the extremities of the object are derived. In particular, the rather general mathematical approach has been based on a strategy already proposed by some of the authors in a previous work, and here extended to anisotropic hollow phases. The key of the involved method is to reduce the differential Boundary Value Problem (BVP) to the equivalent linear algebraic one, by means of a special matrix-like arrangement of the governing equations and invoking the Complex Potential Theory for anisotropic materials. The obtained general solution has been then easily particularized to the two phase FGMC representing the circular concrete column confined by FRP sheets. In the second part of the paper, the above mentioned solutions for the concrete column are "moved" within the post-elastic range and we investigate the evolution of the stress field in the solid components when the concrete core is characterized by an Intrinsic Curve or Schleicher-like elastic–plastic behavior endowed with associate flow rule, and the FRP cylindrically orthotropic hollow phases obey to an elastic–brittle Tsai–Hill anisotropic yield criterion. At the end, the elastic and post-elastic response of the overall solid and predictive formulae for estimating the failure mechanism, in terms of concrete ultimate compressive strength, confining pressure and strain at failure, are derived. The obtained results are finally compared with several experimental literature data, highlighting the very good agreement between the analytical predictions and experimental tests.

0. Introduction

Functionally Graded Materials (FGMs) are mostly treated as non-homogeneous materials with elastic moduli and mechanical parameters that vary continuously or in a piece-wise continuous manner along at least one spatial direction. In this framework, the present work will refer to layered circular cylinders constituted by n anisotropic and homogeneous hollow phases and a central isotropic core, here named n-plies Functionally Graded Material Cylinders (n-FGMCs).

Cylindrical shells are often used as basic structural components in engineering applications. Much research has been conducted on isotropic or laminated composite plates and shells, with reference to thermo-elastic problems of functionally graded infinite hollow cylinders. In particular, Liew et al. [1] obtained analytical solutions of a functionally graded circular cylinder by a novel limiting process that employs the solutions of homogeneous circular hollow cylinders. Shao [2] derived analytical solutions for the mechanical stresses of a functionally graded circular cylinder with finite length, finding mechanical and thermal stresses for the two-dimensional thermo-elastic problems, where the cylinder is assumed to be composed of n homogeneous fictitious layers in the radial direction. There, only special axis-symmetrical load conditions with simply supported boundary conditions at the two ends of the object are considered, and the solutions are found by means of specific trigonometric series. Mian and Spencer [3] determined some results for isotropic laminated FGMs with specific variation of the elastic moduli in the direction of the axis of the object. Recently, Fraldi and Cowin [4] obtained further new results for non-homogeneous and anisotropic materials, including FGMs, by using a *Stress-Associated Solution* theorem. Also, Alshits and Kirkhner [5] derived some elastic solutions

* Corresponding address: Department of Structural Engineering, University of Napoli "Federico II", via Claudio 21, 80125, Napoli, Italy. Tel.: +39 081 768 3731; fax: +39 081 768 3332.

E-mail address: fraldi@unina.it (M. Fraldi).

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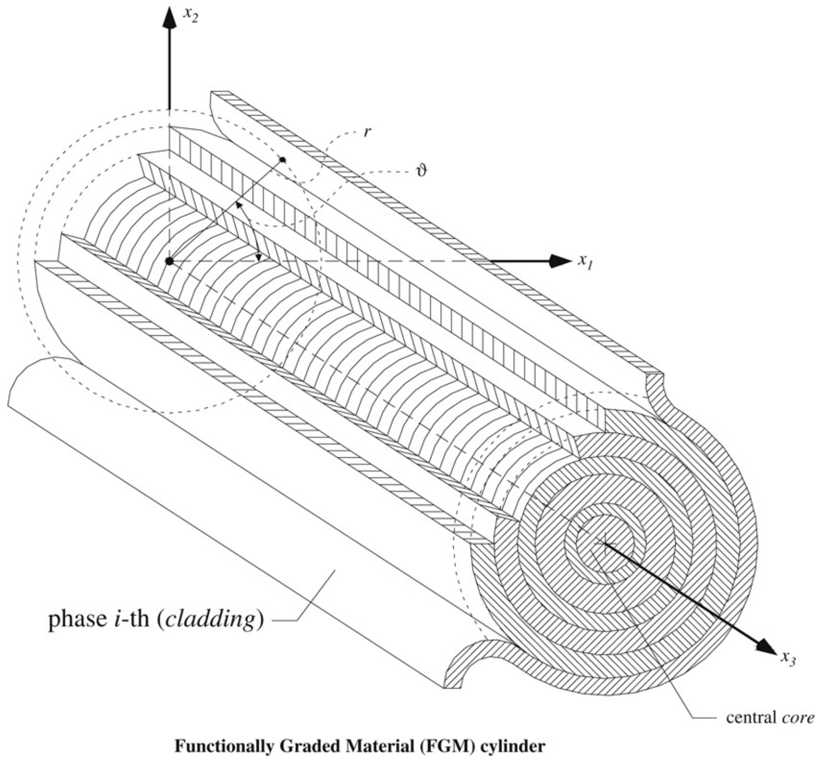

Fig. 1. Sketch of the n-plies functionally graded material cylinder.

for radially inhomogeneous and cylindrically anisotropic circular cylinders, where no variations of the stresses along the axis of the cylinder are assumed. Other interesting results for laminated composite tubes were obtained by Chouchaoui and Ochoa [6], Chen et al., [7], Tarn [8] and Tarn and Wang [9]. In particular, these last two authors, by employing the so-called *state space approach*, starting from some results obtained by Lekhnitskii [10] and involving an original rearrangement of the field equations that yields to isolate new *state variables*, built up analytical solutions for elastic problems in which the stresses do not vary along the axis of the composite tube. Although under the hypothesis of *generalized plane strain and torsion*, this strategy offers the possibility to find exact solutions for laminated composite tube under extension, torsion, shearing and pressuring, by assuming cylindrical anisotropy for each phase. Huang and Dong [11] presented a procedure for the analysis of stresses and deformations in a laminated circular cylinder of perfectly bonded materials with the most general form of cylindrical anisotropy and Ting [12] furnishes an useful strategy for uncoupling equations in cylindrically anisotropic cylinders, to which the authors will refer in the following.

To the authors' knowledge, only a limited amount of work has been carried out on closed-form elastic and post-elastic solutions for anisotropic FGM cylinders. To this purpose, in Section 1 is developed a mathematical procedure aimed at constructing analytical elastic solutions for FGMCs, constituted by n cylindrical hollow phases and a central core, each of them modeled as linearly elastic, homogeneous and isotropic (Fig. 1). Under the hypotheses of axis-symmetrical boundary conditions and perfectly bonded phases, analytical solutions for self-equilibrated axial forces applied at the extremities of the object are derived for the general case, that is for any number of phases and for arbitrary elastic moduli. The approach is based on a specific choice of Love's biharmonic scalar functions $\chi^{(i)}(r, x_3)$, [13], where (i) stands for the generic i-th phase. As already proven by some of the authors in a previous work [14] with reference to a more general case, the field differential equations involving partial derivatives reduce to a set of Euler-like ordinary one-dimensional uncoupled differential equations, that are suitable to be solved by means of an *in cascade* technique. As a consequence, the continuity conditions for displacements and stresses at the interfaces become algebraic: the whole set of boundary conditions can be then written in a matrix form and the problem can be easily solved [14].

In Section 2 the above described strategy is generalized to cylindrically orthotropic FGMC hollow phases, by invoking the classical Complex Potential Theory for anisotropic materials [10] and using a suitable rearrangement of the equations suggested by Ting [12]. In this way, due to the permanency of the axis-symmetry of the problem, a strategy similar to that constructed for isotropic FGMCs is pursued and the Boundary Value Problem (BVP) is again reduced to an algebraic system, governed by a matrix whose order is depending upon the number of hollow phases and whose coefficients are explicitly related to the geometrical and mechanical parameters of the object. Afterwards, in Section 3, in order to facing the cilindrical concrete specimen reinforced by means of FRP sheets this last procedure is specialized to a generic two-phase FGMC, constituted by an isotropic core and by a surrounding hollow cylindrical orthotropic phase, by explicitly furnishing the elastic solutions in terms of stress, strain and displacement fields for the case of axial forces applied at the ends of the solid. In Section 4, with the aim of analysing the post-elastic response of the concrete column confined by FRP, the elastic solutions found in Section 3 are extended to the post-elastic range, by investigating the evolution of the stress field when the FGMC core is characterized by an Intrinsic Curve or Schleicher-like elastic–plastic response with associate flow rule and the FRP sheets obey the brittle Tsai–Hill anisotropic yield criterion. The choice of these post-elastic behaviors is suggested by experimental evidence reported in literature for these materials, as well as the cylindrical orthotropy of the hollow phase intrinsically yields to consider several perfectly bonded FRP layers as an equivalent one, interpreting their overall mechanical response by invoking the theory of homogenization and the mechanics of composites [15].