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Response of statically determined steel beams reinforced by CFRP plates in the elastic–plastic regime

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This paper is dedicated to Professor Giulio Maier on the occasion of his birthday.

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ABSTRACT

The paper presents a simple approach to evaluate the response of statically determined steel beams reinforced by carbon fiber reinforced polymer (CFRP) plates in the elastic–plastic regime. The formulation is applied to two cases: simply supported beams both with distributed and concentrated load. The proposed solution is validated by comparison with experimental data available in the literature.

1. Introduction

Civil structures and infrastructures may become structurally inadequate for different reasons: e.g. deterioration of materials, variation of the loads acting on the structure, design errors, etc. Particularly, in the case of steel structures the load bearing capacity may be drastically reduced by fatigue and corrosion damage. Standard techniques of rehabilitation consist mainly of the application of steel plates where the structure is damaged; but, therefore, problems such as steel corrosion still remain and difficulty in fitting complex profiles can arise.

The use of CFRP to repair and rehabilitate damaged steel and concrete structures is continuously increasing due to the well-known high mechanical properties of this material, with particular reference to its very high strength to density ratio, see [1]. For instance the retrofitting of existing beams by bonding a CFRP plate to its soffit has numerous advantages, such as the increase in stiffness and ultimate flexural capacity. Despite its intrinsic cost, the possibility to shape the CFRP lamina and to avoid the cumbersome work associated with the standard rehabilitation techniques and finally the very low dead weight added makes the overall cost for strengthening to be reduced. Many examples from the literature show the effective use of CFRP in civil engineering, see e.g. [2–8].

In such retrofitted structures, collapse may consist of the following scenarios: debonding of the CFRP plate, rupture of the carbon reinforcement and achievement of the maximum flexural capacity of the composite section, see [9]. Debonding of the CFRP plate is one of the most important failure modes, since it prevents the achievement of the full flexural capacity.

Reinforcement of structures with CFRP lamina is a subject of intensive research carried out from theoretical, experimental and applicative standpoints; see recently, e.g.: [10–12]. In this paper we propose a simple approach to evaluate the response, in terms of interface shear stress, CFRP axial force and maximum flexural capacity of statically determined beams reinforced by CFRP strips taking into account the non linear elasto-plastic material behavior. The scope of the present work is to provide a practical and simple approach for the design at the ultimate limit state (ULS) of reinforced structures against the above mentioned potential failure modes.

To this purpose it is necessary to know the mechanical properties governing the interface behavior (e.g. fracture energy, mode I–II resistances). There are several methods in order to correctly estimate these properties: the double strap joint test, see [13–15]; the single strap joint test as in [16,17]; the three point bending test as in [18,19]; the peel test to measure interface properties in case of mode-I loading, see [20], and the pure shear test combined with an inverse approach as proposed in [21].

In the case of steel structures the interaction of delamination process with steel plastic deformation is worth studying, in order to provide guidelines that guarantee that not only the local

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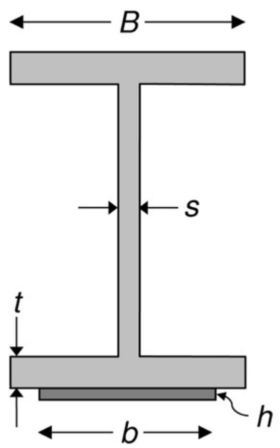

Fig. 1. Assumed geometry of a reinforced I-section.

strength is recovered by the strengthening process, but also a certain degree of ductility is rehabilitated, the last being a topic of high concern in the seismic design, see for instance [22].

The determination of interfacial stresses and CFRP axial force under the assumption of linear elastic response of the materials has already been pursued by different authors, see e.g. [23–25]. For instance these authors have shown that in case of linear elastic response the interface shear stress assumes its maximum value at the reinforcement ends.

The solution proposed typically applies to the case of reinforced steel beams, but it holds in each case where the material behavior is ductile. The main limitations of the solution proposed consists of the fact that it is applicable only to statically determined beams and it is valid only at a certain distance from the reinforcement ends, i.e. where the response of the structure is not influenced by the local effects due to the abrupt termination of the carbon plate, which, however, are already well captured by the above mentioned linear elastic solutions [23–25].

2. Formulation

Once the bending moment distribution acting in the composite beam, say M_{TOT}, has been computed (by simple equilibrium equations since the composite beam is assumed to be statically determined), the response in the elastic–plastic regime, is derived here by simple sectional analysis. The proposed solution is based on the following assumptions:

- linear elastic behavior of CFRP and elastic perfectly plasticity for steel;
- no slippage between steel and CFRP;
- the contribution of the adhesive layer between steel and CFRP to the bending and axial stiffness of the section is negligible;
- the bending stiffness of the CFRP is negligible;
- plane sections remain plane;
- the CFRP axial stress is assumed to be uniform and equal to the value assumed in the center of gravity of the carbon section.

The derivation is here restricted, without loss of generality, to an I-section, see Fig. 1, however it can be extended to other sections of arbitrary shape.

According to the above hypotheses, the equations governing the response of the section are as follows, see Fig. 2:

- Compatibility equation:

$$\epsilon(y) = \eta + y\chi. \tag{1}$$

- Constitutive laws:

$$\sigma_c = E_c\epsilon \tag{2}$$

$$\sigma_s = \begin{cases} E_s\epsilon & \text{if } |\epsilon| \le \dfrac{\sigma_0}{E_s} \\ \sigma_0 & \text{if } |\epsilon| > \dfrac{\sigma_0}{E_s}. \end{cases} \tag{3}$$

- Equilibrium equations:

$$\int_{A_{TOT}} \sigma \,\mathrm{d}A = 0 \tag{4}$$

$$\int_{A_{TOT}} y\sigma \,\mathrm{d}A = M_{TOT} \tag{5}$$

where: subscripts c and s refer to CFRP and steel, respectively; σ and ϵ are the axial stress and strain; E_c, E_s are the elastic moduli of CFRP and steel, respectively, and σ_0 is the steel yield stress.

The CFRP axial force reads:

$$N_c = \int_{A_{\mathrm{CFRP}}} \sigma \,\mathrm{d}A = \sigma_c hb. \tag{6}$$

Moreover, as suggested by Fig. 3, the horizontal equilibrium in the composite lamina imposes that the interface shear stress reads:

$$\tau_i = \frac{1}{b} \cdot \frac{\mathrm{d}N_c}{\mathrm{d}x} \tag{7}$$

where coordinate x starts from the beam support, as visualized in Fig. 5.

In the proposed formulation the interface normal stress is assumed equal to zero; however this is not a restrictive assumption, since the normal stress is significant only at the reinforcement ends, where the present solution does not apply.

2.1. Elastic section

When the material behavior is linear elastic the above equations lead to the transformed section approach, which provides the following formula to compute the CFRP force:

$$N_c = nbh\frac{M_{TOT}}{I_{\mathrm{hom}}}\left(H + \frac{h}{2} - v_G\right) \tag{8}$$

being v_G the distance of the center of gravity of the transformed section from the top of the section, $n = E_c/E_s$ and I_{hom} the second moment of inertia of the transformed section.

Combining Eqs. (7) and (8) it follows that:

$$\tau_i = nh\frac{1}{I_{\mathrm{hom}}}\frac{\mathrm{d}M_{TOT}}{\mathrm{d}x}\left(H + \frac{h}{2} - v_G\right) \tag{9}$$

which coincides with the classical Jourasky solution.

2.2. Partially plastic section

Due to the geometry of the section different cases must be considered depending on the position of the elastic zone with respect to the web and flange of the I-section, see Fig. 2, namely:

- case (1): $\lambda_1 \le t$ and $\lambda_2 \le t$
- case (2): $t < \lambda_1 \le H - t$ and $\lambda_2 \le t$
- case (3): $t < \lambda_1 \le H - t$ and $\lambda_2 > t$
- case (4): $\lambda_1 > H - t$ and $\lambda_2 \le t$