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Generic nonlinear modelling of restrained steel beams at elevated temperatures

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ABSTRACT

This paper presents a novel numerical method to describe the nonlinear behaviour of a restrained steel beam at elevated temperatures, which is based on a non-discretisation semi-analytical formulation of a generic steel cross-section with elastic and plastic parts. The model incorporates moderately large displacement effects as well as material nonlinearity to assess the performance of steel beams exposed to thermal loading in a compartment fire, and it considers the yielding and development of catenary action affects with an increase in the temperature. The effects of the thermal gradient along the length of the beam are considered in the formulation, in which the cross-section is subjected to an arbitrary thermal profile. Degradation of the stiffness and yield strength with increasing temperature is simulated using empirical retention functions given elsewhere, and the technique is shown to be part of the class of boundary value problems in engineering mechanics. The generic technique developed in the paper is shown to agree with solutions given by ABAQUS finite element modelling, and it provides a computationally superior formulation to that of commercial finite element packages, and provides a formulation which may be used as a platform for structural design and evaluation.

1. Introduction

The behaviour of steel beams in buildings at elevated temperatures has become an important research area in structural engineering, and has been investigated by many researchers over recent years in a quest to develop rational and economic procedures of design against fire loading. As a contribution to this research area, the present paper is concerned with an efficient and generic numerical modelling of a steel beam in a frame compartment, which is able to capture the full-range response which hitherto has needed the use of stiffness and flexibility based approaches with interpolation functions in finite element and other discretisation models.

Saab and Nethercot [1] presented a formulation for the nonlinear analysis of two-dimensional steel frames under fire conditions using the finite element method. They considered the temperature–deformation history of heated steel frames, as well as the effects of residual stresses and thermal bowing, on the behaviour of frames subjected to fire. Wang and Moore [2] presented a finite element computer program for studying the structural behaviour of steel frames at elevated temperatures; the important effects of semi-rigid beam-to-column connections were included in their work. A description of the principles of a three-dimensional frame analysis (3DFIRE) which was developed for the purpose of modelling the behaviour of skeletal frames under fire conditions was presented by Najjar and Burgess [3]. This was based on an existing two-dimensional program developed earlier at The University of Alberta for the nonlinear spread of yielding of rigid steel frameworks at ambient temperatures [4], which was extended to a three-dimensional capability that covers geometric and material nonlinearities, including changes to the material properties as the temperature increases. Amongst other applications, this powerful program [4] was used by Bradford and Trahair [5] to investigate inelastic lateral buckling of beam–columns and frames, albeit at ambient temperature. Allam et al. [6] presented a structural fire design strategy for steel frames at elevated temperatures. Using a three-dimensional finite element program, it was shown that the steelwork would, in general, survive to very high temperatures and continue to provide fire resistance, although it may experience large deflections. This conclusion is consistent with the well-reported full-scale benchmark test results observed at the Cardington test facility, which are cited widely.

A methodology for an advanced analysis technique for studying the large-displacement inelastic behaviour of steel frames subjected to localised fire was presented by Liew et al. [7] using one element per member to model each structural component. Zhao [8] proposed a direct iteration method capable of predicting the nonlinear behaviour of steel frames at elevated temperatures. The total structural response corresponding to a specified load level was calculated by a straightforward iteration process using a direct iteration method. A second-order elastic–plastic analysis, as well as a finite element model for the analysis of plane frames in fire

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which was derived from a two-node co-rotational beam element, was presented by Toh et al. [9]. They extended the three classical engineering plasticity theorems, viz. the lower bound theorem, upper bound theorem and the uniqueness theorem to fire analysis in which the lower bound theorem represented the collapse temperature (or time) for a structure under thermal effects and was a lower bound when both the equilibrium and yield conditions were satisfied; however the upper bound theorem postulated that the collapse temperature was an upper bound only when the mechanism condition was satisfied, irrespective of the equilibrium condition. When the three conditions of equilibrium, yield and a mechanism were satisfied, the uniqueness theorem stated that the corresponding collapse temperature was unique.

The effect of strain reversal was incorporated into the finite element formulation for the analysis of two-dimensional frames at elevated temperatures by Tan et al. [10]. Iu and Chan [11] presented a finite element formulation of beam–column elements which was based on a plastic hinge approach to model the elasto-plastic strain hardening material behaviour for steel. Landesmann et al. [12] developed an advanced numerical tool of analysis capable of estimating the inelastic large-displacement behaviour of plane steel-framed structures under fire conditions. They performed a nonlinear transient heat transfer analysis based on the finite element method, following the main guidelines of the European EC3 code [13] for steel structures under fire attack. A nonlinear finite element analysis of axially restrained steel beams at elevated temperatures which employs the axis arc-length and section rotation of the deformed beam as basic variables has been developed recently by Li et al. [14]. They proposed a balance function that measures the error of the equilibrium between the internal and external forces on a cross-section of the beam. Based on a series of stress–strain curves obtained experimentally for various levels of temperatures, an artificial neural network was employed in the material modelling of steel by Hozjan et al. [15]; however the geometric and material nonlinear analysis of plane frame structures subjected to fire was performed using the finite element method.

El-Rimawi et al. [16] developed a secant stiffness method for the analysis of isolated steel beams under the influence of thermal distributions caused by fire. The model included both material and geometric nonlinearities and enabled full planar skeletal frames to be analysed up to failure under local or overall fire conditions. Li et al. [17] proposed a finite element approach for the analysis of the nonlinear behaviour of steel frames subjected to fire where the elasto-plastic stiffness relationship for the elements was established using the generalised Clough model. It is noteworthy that the analysis of structures subjected to fire was converted to the analysis of structures subjected to unbalanced forces induced by a temperature increment. Iu et al. [18,19] investigated material yielding caused by both axial and bending spring stiffnesses in a fire in their hinge model; they presented a nonlinear material model representation allowing for the interaction of both axial force and bending moment which was based on the plastic hinge method. It was shown that the axial spring stiffness used in conjunction with the plastic hinge method is effective in describing the structural behaviour of beams at elevated temperatures.

An elasto-plastic analysis of semi-rigid steel frames subjected to elevated temperature was presented by Vimonsatit et al. [20], in which an updated Lagrangian approach was used within a discretised structural framework to incorporate the nonlinear geometric effects on the equilibrium and constitutive conditions. Modelling of elasto-plastic behaviour was based on the generalised plastic hinge concept for beam–column elements which obeyed piecewise linear yield criteria. More recently, a beam–column model for simulating the inelastic behaviour of three-dimensional steel frames was reported by Ma and Liew [21], in which yielding of the section was modelled by two nested stress-resultant surfaces, and it considered the softening effect of the steel material at elevated temperatures. Li and Wang [22] developed a spline finite beam element for the analysis of axially restrained steel members subjected to elevated temperatures in fire. Governing equations for the beam were established according to the internal force and external force equilibrium at each node of the elements representing the beam, and the nonlinear equation system was solved by the Newton continuation method.

The present paper deals with a generic model for the nonlinear behaviour of an isolated steel beam at elevated temperatures in a frame compartment. The beam is restrained at its two ends with translational and rotational springs (which represent semi-rigid joints) and is subjected to an arbitrary thermal profile which varies over the cross-section as well as along the length of the beam. The model considers the effects of yielding of the material, as well as catenary action in the member which occurs at high temperatures, and is validated against ABAQUS solutions. It is then utilised to elucidate some significant factors on the response of beams in a compartment fire.

2. Elevated-temperature material properties of steel

Based on fire tests under either transient or steady-state conditions of heating, a number of empirical models have been proposed for the material properties of steel at elevated temperature. The ECCS [23] recommends a simplified tri-linear model in which the creep strain is assumed to be included implicitly. At a specific thermal state, this model of the stress–strain curve depends on four material parameters, viz. the initial modulus of elasticity, the proportional stress limit, the yield stress and the softening modulus of elasticity. The EC3 [13] relationship is more complicated than that of the ECCS, insofar as it attempts to provide a fit for the various portions of the stress–strain curve, using seven linear and parabolic equations to represent the stress–strain (σ–ε) curve including the strain-hardening region. Using a Ramberg–Osgood formulation, the strains may be expressed as a function of the stresses in the form

$$\varepsilon = \frac{\sigma}{E_{s,T}} + \alpha \left(\frac{\sigma}{E_{s,T}} \right)^n, \tag{1}$$

where $E_{s,T}$ is the initial tangent stiffness of the stress–strain curve at elevated temperature, and α and n are curve fitting coefficients. A set of values for α and n at elevated temperatures has been given by Saab and Nethercot [1].

Lie [24] proposed a useful bilinear curve, with a small transition between the linear portions, to represent the stress–strain curve at elevated temperatures. The curve consists of two separate equations as

$$\sigma = \begin{cases} E_{s,T} \cdot \varepsilon & \varepsilon \le \varepsilon_P \\ (c_1 \varepsilon + c_2)\, \sigma_{y,T} - \dfrac{c_3 \sigma_{y,T}^2}{E_{s,T}} & \varepsilon > \varepsilon_P \end{cases} \tag{2}$$

in which

$$\varepsilon_P = \frac{c_2 \sigma_{y,T} - c_3 \sigma_{y,T}^2 / E_{s,T}}{E_{s,T} - c_1 \sigma_{y,T}} \tag{3}$$

and where $\sigma_{y,T}$ is the yield stress at elevated temperature. The first part of Eq. (2) describes the linear elastic portion, and the second describes the inelastic portion of the steel material, in which the coefficients are taken as $c_1 = 12\cdot5$, $c_2 = 0\cdot975$ and $c_3 = 12\cdot5$. The temperature-dependent elastic modulus and yield strength are given in many standards; that given by the Australian AS4100 [25]