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Higher Order Coherences for fatigue crack detection

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ABSTRACT

Higher Order Coherences (normalized Higher Order Spectra) are the tools to identify the relationship between the different harmonic components in a signal. The vibration of a structure having a crack also generates several harmonics of the exciting frequency due to its breathing (closing and opening) behaviour which is a non-linear phenomenon. Presently two types of the HOC — namely, the Bicoherence and the Tricoherence, are used for the fatigue crack detection. This paper presents the observations made on the HOC on the numerically simulated experiment of a cantilever beam with and without cracks. The robustness of the HOC in the crack detection even for the noisy response vibration data has also been brought out.

1. Introduction

Fatigue cracks are often present in structures which are subjected to repeated loads. The detection of such fatigue cracks in structures has been researched for decades. Several research studies have been carried out. Doebling et al. [1] gave a review of the research on crack and damage detection in structures using vibration data. The detection methods are predominately based on the change in modal properties (natural frequencies and/or mode shapes). In some recent studies, the wavelet analysis is applied to the structural responses and/or mode shapes to detect the crack. Kim and Melhem [2] gave the review of the research conducted on the damage detection by wavelet analysis. Most of these methods are suggested mainly based on the numerically simulated examples, and they may require controlled experiments and the measurements of both responses and the force applied for the excitation of the structures in practice.

However a method would be well-appreciated and used in the field, if the method is capable of detecting the fatigue crack from the vibration responses only either due to known given excitation or due to the natural excitation like the rotating speed of machines or wind excitation for many structures, etc. One such approach is the use of Bicoherence for the detection of a crack [3]. The Bicoherence is a type of Higher Order Coherence (HOC), and is the normalized bispectrum to scale the amplitude between 0 and 1 [4,5]. The Bispectrum is the double Fourier Transformation of the third order moment of a time signal that involves two frequency components (both amplitudes and phases) of the signal with a third frequency component summation of first two frequencies. Since the crack in a structure generates higher harmonics of the exciting frequencies due to the breathing of the crack during vibration. This behaviour is predominant if the exciting frequency is close to the natural frequency. The use of other HOC namely, Tricoherence has also been investigated here for the crack detection. The Tricoherence is the Fourier transformation of the fourth order moment of a signal that relates four frequency components (three frequency components with a frequency component summation of the three frequencies). Hence, in the present study, the usefulness and robustness of both the Bispectrum and the Tricoherence have been accessed for the crack detection. These HOCs have been applied on the numerically simulated experiment of a cantilever beam with and without cracks.

The computation of the HOCs at all frequencies may be time consuming so here the computation has been carried out at elemental level. It means that the HOCs are calculated just for the exciting frequency and its related harmonics only because breathing of the crack generates harmonics in the exciting frequencies. It will save the computational effort and time without losing any information related to the crack detection process. The paper highlights the results observed, and the robustness of the HOC in crack detection has also been brought out.

2. The HOC

The nth order moment function of a signal, $x(t)$ is defined as,

$$\mathbf{R}_{xxxx\ldots x}(\tau_1, \tau_2, \tau_3, \ldots, \tau_n) = E[x(t)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)\ldots x(t-\tau_n)], \quad (1)$$

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where $E[.]$ denotes the expectation operator, and τ as delay. The Power Spectral Density (PSD) is defined as the Fourier Transformation (FT) of a 2nd order moment function of Eq. (1), and is computed as

$$PSD, \mathbf{S}_{xx}(f_k) = E[\mathbf{X}(f_k)\mathbf{X}^*(f_k)], \qquad k = 1, 2, 3, \ldots, N \tag{2}$$

where $\mathbf{S}_{xx}(f_k)$ is the PSD, $\mathbf{X}(f_k)$ and $\mathbf{X}^*(f_k)$ are the DFT and its complex conjugate at frequency f_k for the time series $x(t)$. N is the number of the frequency points. $E[.]$ denotes the mean operator here. Let us assume that the time domain signal, $x(t)$, of the time length equals to t. This time signal has been divided into n number of segments with some overlap and each segment contains $2N$ number of data points with sampling frequency, f_s Hz. If $X_r(f_k)$ is the FT of the rth segment, $x_r(t)$, at the frequency, f_k, then the averaged or mean PSD can be computed as

$$\mathbf{S}_{xx}(f_x) = \frac{\sum_{r=1}^{n} X_r(f_k)X_r^*(f_k)}{n}, \tag{3}$$

where $f_k = (k-1)\mathrm{d}f$, $\mathrm{d}f = \frac{f_s}{2N}$.

The PSD gives only the content of different frequencies and their amplitudes in a signal. However, the HOS – Bispectrum and Trispectrum provide insights into non-linear coupling between frequencies (as it involves both amplitudes and phases) of a signal compared to the traditional PSD. For example, the Bispectrum is the double FT of a 3rd order moment of a time signal [4,5] that involves two frequency components (both amplitudes and phases) of the signal together with a frequency component summation of first two frequencies, and is mathematically expressed as

$$\mathbf{B}_{xxx} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathbf{R}_{xxx}(\tau_1, \tau_2)\mathrm{e}^{-j2\pi(f_1\tau_1+f_2\tau_2)}\mathrm{d}\tau_1\mathrm{d}\tau_2, \tag{4}$$

where $\mathbf{R}_{xxx}(\tau_1, \tau_2) = E[x(t)x(t-\tau_1)x(t-\tau_2)]$ is the 3rd order moment, and the Bispectrum is computed by the signal DFT as

$$\text{Bispectrum, } \mathbf{B}_{xxx}(f_l, f_m) = E[\mathbf{X}(f_l)\mathbf{X}(f_m)\mathbf{X}^*(f_l+f_m)], \quad f_l + f_m \le f_N \tag{5}$$

$$\mathbf{B}_{xxx}(f_l, f_m) = \frac{\sum_{r=1}^{n} \mathbf{X}_r(f_l)\mathbf{X}_r(f_m)\mathbf{X}_r^*(f_l+f_m)}{n}. \tag{6}$$

The Bispectrum is complex and interpreted as measuring the amount of coupling between the frequencies at f_l, f_m, and $f_l + f_m$, and is described by 'quadratic phase coupling'.

Similarly the Tricoherence is defined as triple FT of a 4th order moment of a time signal, and is computed as

$$\text{Trispectrum, } T_{xxxx}(f_l, f_m, f_p) = E[X(f_l)X(f_m)X(f_p)X^*(f_l+f_m+f_p)], \quad f_l+f_m+f_p \le f_N \tag{7}$$

$$T_{xxxx}(f_l, f_m, f_p) = \frac{\sum_{r=1}^{n} X_r(f_l)X_r(f_m)X(f_p)X_r^*(f_l+f_m+f_p)}{n}. \tag{8}$$

As can be seen from Eq. (7), the Trispectrum relates the responses at three frequency components (f_l, f_m, f_p) of a signal with a response component equal to the sum of these three frequencies $(f_l + f_m + f_p)$. Hence the relation between the different harmonic components of a signal can be observed by this Trispectrum. The HOS is generally normalized to scale the amplitude between 0 and 1 is called the High Order Coherences (HOC). A commonly used normalization method for both the Bispectrum and Trispectrum is used here as defined below [5],

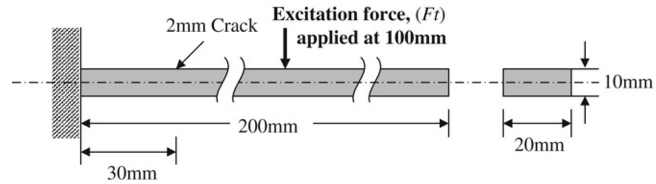

Fig. 1. The beam with a crack.

$$\text{Bicoherence, } B_{lm}(f_l, f_m) = b^2(f_l, f_m) = \frac{|\mathbf{B}_{xxx}(f_l, f_m)|^2}{E\left[|\mathbf{X}(f_l)\mathbf{X}(f_m)|^2\right]E\left[|\mathbf{X}(f_l+f_m)|^2\right]} \tag{9}$$

$$B_{lm}(f_l, f_m) = b^2(f_l, f_m) = \frac{|\mathbf{B}_{xxx}(f_l, f_m)|^2}{\left(\frac{\sum_{r=1}^{n}|\mathbf{X}_r(f_l)\mathbf{X}_r(f_m)|^2}{n}\right)\left(\frac{\sum_{r=1}^{n}|\mathbf{X}_r(f_l+f_m)|^2}{n}\right)} \tag{10}$$

$$\text{Tricoherence, } T_{lmn}(f_l, f_m, f_p) = t^2(f_l, f_m, f_p) = \frac{\left|\mathbf{T}_{xxxx}(f_l, f_m, f_p)\right|^2}{E\left[\left|\mathbf{X}(f_l)\mathbf{X}(f_m)\mathbf{X}(f_p)\right|^2\right]E\left[\left|\mathbf{X}(f_l+f_m+f_p)\right|^2\right]} \tag{11}$$

$$T_{lmn}(f_l, f_m, f_p) = t^2(f_l, f_m, f_p) = \frac{\left|\mathbf{T}_{xxxx}(f_l, f_m, f_p)\right|^2}{\left(\frac{\sum_{r=1}^{n}\left|\mathbf{X}(f_l)\mathbf{X}(f_m)\mathbf{X}(f_p)\right|^2}{n}\right)\left(\frac{\sum_{r=1}^{n}\left|\mathbf{X}(f_l+f_m+f_p)\right|^2}{n}\right)}. \tag{12}$$

The concept of correlating different harmonic components (both amplitudes and phases) in the Higher Order Coherences clearly indicates that they are the non-linear estimators. The normalization process of the HOS limits the scale of the HOC from 0 to 1, where the scale 0 indicates no relation among frequency components and the perfect relations when the scale equals 1.

3. Simulated examples

In simulations, an example of a cantilever beam with the geometrical properties — Length, 200 mm and cross-section 10 mm × 20 mm, and the material properties — Elasticity, 210 GN/m^2 and density 7800 kg/m^3. An FE model of the cantilever beam was constructed using two node Euler–Bernoulli beam elements. A consistent mass matrix is used. Each node has two degrees of freedom, namely the translational displacement and bending rotation. The model is shown in Fig. 1. The damping of 0.5% was assumed at the first mode, and the stiffness proportional damping matrix was constructed. The model has 10 elements and 20 degrees of freedom as the node 1 for the cantilever beam has zero degrees of freedom. The first bending mode has been estimated as 209.45 Hz. A typical FE Model is shown in Fig. 2.

To this FE model, a crack of 20% (i.e. 2 mm in 10 mm depth of the beam cross-section) was introduced at $x = 30$ mm from the fixed end of the cantilever beam (i.e., at the centre of the 2nd element), which has been modelled using the method suggested by Sinha et al. [6] for a crack in a beam that utilized the triangular reduction in local flexibility (EI) in a beam element having a crack as shown in Fig. 2(b) to simulate the open crack. The first bending mode has now been reduced to 197.72 Hz when crack was fully opened. Sinha and Friswell [7] have also demonstrated the use of this crack modelling approach in the crack breathing (open and closed)