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Iterative system buckling analysis, considering a fictitious axial force to determine effective length factors for multi-story frames

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ABSTRACT

Traditional elastic buckling analysis, based on the system buckling approach, is a convenient tool for the evaluation of effective length factors of columns, in the stability design of multi-story frames. This method is superior to other analytical approaches, such as the isolated subassembly and story-based approaches, in that the inter-story and inter-column interactions are inherently taken into account. Nevertheless, use of the conventional critical load expression, in combination with results of elastic buckling analysis, may yield an excessively large effective length in members having relatively small axial forces. The present paper proposes an iterative elastic buckling analysis to determine reasonable effective length factors of columns in multi-story frames. In this paper, numerical procedures for an iterative buckling analysis using a modified geometric stiffness matrix, are described to obtain the effective length factors of the columns in multi-story frames. The axial force term in the geometric stiffness matrix is modified by adding a fictitious axial force to make the columns buckle along with the overall buckling of the frame. Iterative eigenvalue analysis is performed using the modified geometric stiffness matrix, to obtain the effective length factors of each column using the critical load expression. Example frames presented in this paper demonstrate that the proposed method not only provides excellent outcomes by amending the weakness associated with traditional elastic buckling analysis for determining the effective length factor, but is also a competitive alternative in the design of multi-story frames.

1. Introduction

The concept of an effective length has been used in conjunction with alignment charts and alternative design equations in the column design of steel frames. There are three approaches to calculate the effective length of compression members: the isolated subassembly approach, the story-based approach, and the system buckling approach [1]. The isolated subassembly approach is the traditional method used to assess the rotational end restraints of an isolated column, which represent the joint stiffness ratio (G-factor) between the column and beam. In spite of its popularity, the isolated subassembly approach has major limitations, in that it makes use of several idealizations that do not match the real structural behavior of steel frames [2–4]. Cheong-Siat-Moy [5] explained the existence of multiple K-factors in the design of leaning columns, and pointed out that the buckling strength of these members should be determined, considering not only the boundary conditions between members, but also the overall system buckling behavior. Bridge and Fraser [6] proposed the use of the linearized stability functions with an iterative procedure, in which a sketch of a buckled mode of the structure is used to improve the accuracy of the G-factor. Aristizabal-Ochoa [7] suggested closed form formulas for evaluating effective length factors in any type of steel frame, including braced, unbraced, and partially braced frames. Similarly, Hellesland and Bjorhovde [8] introduced a method of means that is based on the solution of the isolated subassembly approach, and proved the validity of their proposed method, by comparing it to the numerical solution. Gantes and Mageirou [9] proposed improved analytical expressions for multi-story sway frames with idealized springs. They provided rotational stiffness expressions of a column for different boundary conditions, with respect to the existence of axial forces. In addition, some effort has been devoted to improve the accuracy of the isolated subassembly approach with leaning columns [10,11].

The story-based approach is another alternative used to determine the effective length factors of steel frames. Unlike the isolated subassembly approach, it is assumed that the shear force from a column transfers to the other columns in the same story. Therefore, the stiffer columns in a story support the weaker ones against sidesway buckling, and the sidesway buckling resistance of the story is assumed to be equal to the sum of the buckling resistance of all columns within the story [12]. LeMessurier [13,14] first provided comprehensive discussion for story stability

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behavior, and proposed a practical approach for obtaining effective lengths of columns to consider the story stability behavior. Roddis et al. [15] performed parametric studies by changing the width of a frame bay, moment of inertia, condition of loadings, and height of the column, and concluded that the story-based approach gives more accurate effective length factors than the isolated subassembly approach. Recently, Xu and Wang [16] proposed the analytical expression of the story-based approach, considering initial geometric imperfections of steel frames in order to clarify the effects of the out-of-straightness of columns and out-of-plumbness of frames on the column strength.

Researchers and engineers have started to utilize personal computers to analyze the stability of frame structures based on the system buckling approach with elastic buckling analysis [1,9,17,18], global instability with second-order analysis [19–23] and inelastic buckling analysis [24]. Aimed at obtaining the effective lengths of columns in multi-story frames, the system buckling approach with elastic buckling analysis has been cited as an accurate method in several publications because the inter-story and inter-column interactions are inherent in this approach [9,17,25]. Nevertheless, use of the conventional critical load expression in combination with results of the system buckling approach, may yield excessively large effective length factors for columns that exert small axial forces at the buckling of the overall system [1,12,26]. Furthermore, the effective length factor calculations performed in the system buckling approach may be reasonable only for the columns in the story that are weakest with respect to the sidesway buckling in the system [3].

The main objective of this paper is to propose a method to overcome the weaknesses inherent in traditional elastic buckling analysis, based on the system buckling approach for the design of multi-story frames. The theoretical background of the system buckling approach, and other analytical approaches, are presented briefly in the first part of the paper. Next, the problem of applying the critical load expression, in combination with results of the system buckling approach, is clarified by comparing the results with those from analytic approaches for an example of a seven-story frame. Subsequently, numerical procedures for an iterative buckling analysis using a modified geometric stiffness matrix are described to obtain the effective length factors of the columns. The axial force term in the geometric stiffness matrix is modified by adding a fictitious axial force to make the columns buckle, along with the overall buckling of the frame. The converged effective length factors of the columns are determined based on an iterative eigenvalue analysis, and a continuously modified geometric stiffness matrix. Two-story, three-story and six-story plane frames with different loading conditions are selected as benchmark problems to verify the proposed modification of the system buckling approach. The effective length factors for these examples are computed using the proposed method, and compared with those found using established methods.

2. Methods for evaluating effective length factors

2.1. Isolated subassembly approach

The isolated subassembly approach, also known as the alignment chart method or the *G*-factor method, is characterized by the fact that the rotational restraint provided by the beams at the end of an isolated column is a function of the rotational stiffnesses of the members at the joint of the column.

Based on the several assumptions [27], the governing equations for the braced and unbraced cases can be derived from the slope-deflection equations [28].

$$\frac{G_A G_B}{4}(\pi/K)^2 + \left(\frac{G_A+G_B}{2}\right)\left[1-\frac{\pi/K}{\tan(\pi/K)}\right] + \frac{2\tan(\pi/2K)}{\pi/K} - 1 = 0 \tag{1}$$

$$\frac{G_A G_B(\pi/K)^2 - 36}{6(G_A+G_B)} - \frac{\pi/K}{\tan(\pi/K)} = 0. \tag{2}$$

The relative stiffness factors (*G*-factors) in Eqs. (1) and (2) are defined as

$$G_A = \frac{\sum_A (EI/L)_c}{\sum_A (EI/L)_g} \quad \text{and} \quad G_B = \frac{\sum_B (EI/L)_c}{\sum_B (EI/L)_g} \tag{3}$$

where the subscripts A and B indicate the joints for the column, and the subscripts c and g indicate the terms for the column and girder.

2.2. Story-based approach

The isolated subassembly approach assumes that the shear force of a column does not transfer to other columns in the same story. Therefore, each of the columns in any story buckles in a sidesway mode, independent of the adjacent columns. This assumption may be inadequate in some real cases, such as a structural system with leaning columns, or with frames in two orthogonal directions [29]. The practical application of the story-based approach accounts for the interaction effect between the columns in a story.

Two different methods are recognized in the context of AISC-LRFD [28]: the story-buckling method and the story-stiffness method. The story-buckling method uses the isolated subassembly approach to determine the buckling load of the columns. The effective length factor is determined using Eq. (4).

$$K = \sqrt{\frac{1}{P}\left(\frac{\pi^2 EI}{L^2}\right)\frac{\sum P}{\sum P_{cr}}} \tag{4}$$

where P is the compression force of the member, and the terms $\sum P$ and $\sum P_{cr}$ are the sum of the compression forces of all columns in the story, and the sum of the buckling loads.

The story-stiffness method utilizes lateral displacements of the story instead of the sum of the buckling loads. The effective length factor is given by Eq. (5).

$$K = \sqrt{\frac{1}{P}\left(\frac{\pi^2 EI}{L^2}\right)\frac{\Delta_{oh}\sum P}{0.822\sum(HL)}} \tag{5}$$

where H, L, and Δ_{oh} are the lateral forces on the story, the length of the column, and the lateral displacement of the story. The lateral forces and displacements are determined by first-order lateral load analysis. The lateral loads applied in this analysis are equal to 0.005 times the total gravity load at each story of the frame [1,12,29].

2.3. System elastic buckling approach

The column buckling loads and their corresponding effective length factors for the entire structural system can be determined by system elastic buckling analysis, which uses a numerical eigenvalue computation. The governing equation of system elastic buckling analysis can be expressed as

$$([K_E] + \kappa[K_G])\{\phi\} = \{0\} \tag{6}$$

where $[K_E]$ and $[K_G]$ are the elastic and geometric stiffness matrices of the overall structure, respectively. The term κ is the set of eigenvalues, the smallest one representing the critical value of the system. The term $\{\phi\}$ is an eigenmode vector. The elastic and geometric stiffness matrices for the frame element can be derived