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Simplified solutions for the stress transfer in concrete beams bonded with FRP plates

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Abstract

Strengthening of concrete beams by the external plate bonding technique has become an important research area over the last two decades, particularly with the increasing applications of the advanced fibre reinforced composites. Unlike the unstrengthened beams, the bonded structure shows an undesirable failure of plate debonding. It has been well recognized that central to this failure mode is the stress concentration at the plate end. This paper reports a novel simplified interfacial stress formulation for the FRP bonded concrete beams subjected to arbitrary loadings. It is featured as a compact closed-form formulation with improved accuracy. The solution satisfies the condition of shear stress free at the plate ends and can predict a non-uniform transverse normal stress over the adhesive layer, as the adhesive is treated as a 2-D medium. Numerical validations are conducted to compare the simplified solutions with those obtained from more rigorous formulations, alternative approximate solutions and experimental results. Adequate accuracy and applicability are achieved. This formulation can be easily incorporated in the strength models to predict debonding loads.

Keywords: FRP; Concrete beams; Strengthening; Interfacial stresses; Stress concentration

1. Introduction

As a nation's infrastructure ages, one of the major challenges the construction industry has to face is that the number of deficient structures continues to grow. The applications of using externally bonded steel plates or fibre reinforced polymer (FRP) laminates to reinforce concrete (RC) structures have shown that the technique is sound and efficient and offers a practical solution to this pressing problem. It has been realized that central to the reinforcement effect of externally bonded concrete structures is the transferring of stresses from the concrete to the external reinforcement, which causes the undesirable premature and brittle failure modes, such as debonding initiated from the ends of the bonded plates [1]. A good understanding of this problem is thus important for the development of suitable strength models. Experimental studies usually become the first choice to investigate the problem [2]. However, due to the well-known difficulty in measuring the shear stress directly, a typical approach is to glue strain gauges at the external surface of the bonded plate and calculate the shear stress through the differential strains of the consecutive gauges [3–5]. This method renders two problems: (1) the shear stress is only the average stress between the two associated strain gauges, which does not reflect the real stress distribution when the stress gradient is very high near the plate end; (2) the strain readings comprise the strains caused by both axial extension and bending in the plate, which is not the actual strains occurring at the plate–adhesive interface. Apart from these problems, it is extremely difficult to measure the normal stress/strain using the conventional approach. To the authors' best knowledge, there is no published data of the transverse normal stress recorded by experiments. Numerical methods, including the finite element method (FEA), undoubtedly provide a versatile method to predict the stress distribution [6]. Nevertheless due to the stress

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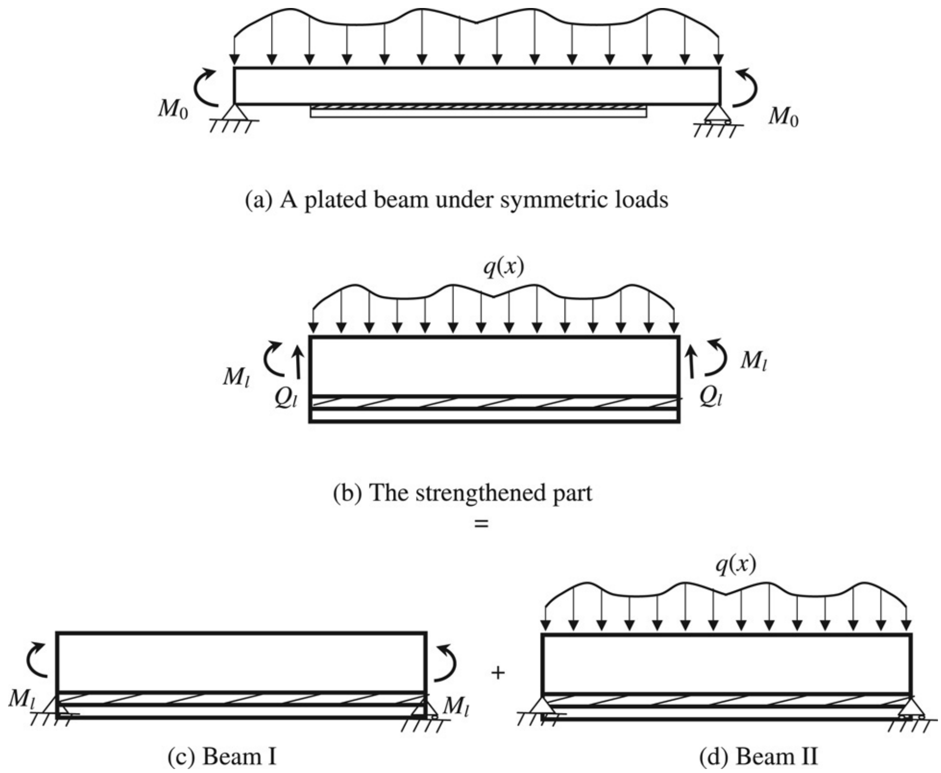

Fig. 1. Diagrammatic representation of the strengthened beam subjected to symmetric loadings.

singularity, it is very time-consuming to refine the mesh near the plate end to obtain the convergent results. As a result there is a need to develop a closed-form solution to predict the stresses, in particular the interfacial stresses, in the bonded RC beam. The early work used the shear-lag type approach [7–9], which was initially developed for the lap-joint by Volkerson [10]. In their solutions, only shear stresses were studied and the equally significant transverse normal stresses were ignored. To overcome this, the deformation compatibility-based approaches were developed in various degree of complexity. Smith and Teng [11] reviewed most of this type of approaches [12–15] and further proposed their own solution avoiding the existing pitfalls. One of typical natures of this type of solutions is that both shear stress and normal stress are assumed uniform over the adhesive layer which naturally leads to the dissatisfaction of the shear stress free condition at the plate ends. Rabinovich and Frostig [16] proposed a high-order solution in which the adhesive is treated as 2-D medium so that the shear stress free condition is satisfied. Unfortunately the results can only be obtained numerically. Shen et al. [17] developed a similar high-order solution using the complement energy method and their solutions are in closed form. Yang et al. [18] extended the work to the bonded beam subjected to arbitrary loadings by superposition of symmetric and anti-symmetric load cases. However, their solutions still have some problems such as the transverse normal stress at the plate–adhesive (PA) interface does not reflect the real distribution. Yang and Ye [19] re-examined the case and proposed an improved rigorous solution that overcomes most of the problems. However, the rigorous solution is so complex that it can hardly be used in the engineering practice. A simplified version of the solution that keeps sufficient accuracy and has the same applicability is obviously required.

This paper reports a novel simplified interfacial stress formulation for the FRP bonded RC beams subjected to arbitrary loadings. The formulation is established on the basis of the simplified form of the rigorous solution [19] and the introduction of the concept of develop length. It is featured as a compact formulation with adequate accuracy and applicability. This paper also validates the simplified formulation through comparisons with the rigorous solutions, the approximate solutions proposed by other researchers and the test results published by Fanning and Kelly [4].

2. Simplified interfacial stresses under end moments

2.1. Basic rationale for the simplified solution and the derivation procedures

Yang and Ye [19] proposed a rigorous elastic solution for a bonded RC beam subjected to uniformly distributed load (UDL) and a pair of equal end moments. The solution for a beam subjected to end moments only is presented in the Appendix.

For a strengthened RC beam subjected to symmetric loadings [see Fig. 1(a)], if only the strengthened part is studied, the applied loads introduce two groups of internal forces at the cut-off cross sections, i.e. a pair of equal bending moments and shear forces [Fig. 1(b)]. The structural system of the strengthened part is equivalent to the superposition of two