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Engineering Structures 30 (2008) 2517–2527

ENGINEERING STRUCTURES

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Slope-deflection equations for stability and second-order analysis of Timoshenho beam–column structures with semi-rigid connections

J. Dario Aristizabal-Ochoa*

School of Mines, National University, A.A. 75267, Medellin, Colombia

Received 24 December 2007; received in revised form 28 January 2008; accepted 12 February 2008
Available online 26 March 2008

Abstract

A new set of slope-deflection equations for Timoshenko beam–columns of symmetrical cross section with semi-rigid connections that include the combined effects of shear and bending deformations, and second-order axial load effects are developed in a classical manner. The proposed method that also includes the effects of the shear component of the applied axial force as the member deflects laterally (Haringx Model) has the following advantages: (1) it can be utilized in the stability and second-order analyses of framed structures made up of Timoshenko beam–columns with rigid, semi-rigid, and simple end connections; (2) the effects of semi-rigid connections are condensed into the slope-deflection equations for tension or compression axial loads without introducing additional degrees of freedom and equations; (3) it is more accurate than any other method available and capable of capturing the phenomena of buckling under axial tension forces; and (4) it is powerful, practical, versatile and easy to teach. Analytical studies indicate that shear deformations increase the lateral deflections and reduce the critical axial loads of framed structures made of members with low shear stiffness. The effects of shear deformations must be considered in the analysis of beam–columns with relatively low effective shear areas (like laced columns, columns with batten plates or with perforated cover plates, and columns with open webs) or with low shear stiffness (like elastomeric bearing and short columns made of laminated composites with low shear modulus G when compared to their elastic modulus E) making the shear stiffness GA_s of the same order of magnitude as EI/L^2. The shear effects are also of great importance in the static, stability and dynamic behavior of laminated elastomeric bearings used for seismic isolation of buildings. Four comprehensive examples are included that show the effectiveness of the proposed method and equations.

Keywords: Beams; Beam–columns; Large deflections; Nonlinear analysis; Semi-rigid connections; Second-order analysis; Stability; Structural analysis; Timoshenko beam

1. Introduction

The slope-deflection method represents a turning point in the evolution and development of the matrix stiffness method as it is known today (Samuelsson and Zienkiewicz [1]). It was presented in 1915 by Wilson and Maney [2] in a *Bulletin* from the University of Illinois at Urbana-Champaign as a general method to be used in the analysis of beam structures with rigid-joints subjected to transverse loads.

The slope-deflection method may be used to analyze all types of statically indeterminate beams and frames. The classic slope-deflection equations are derived by means of the moment–area theorems considering deformation caused by bending moment only and neglecting those caused by shear and axial forces. Basically, a number of simultaneous equations are formed with the unknowns taken as the angular rotations and displacements of each joint. Once these equations have been solved, the moments at all joints may be determined. The method is simple to explain and apply since it is based on the equilibrium of the joints and members. The classic slope-deflection method is generally taught in the introductory structural analysis courses (Norris and Wilbur [3], Kassimili [4]) and used in the structural design (Salmon and Johnson [5]) because it provides a clear perspective and a complete understanding of how the internal moments and the corresponding deformations are interrelated, both of which are essential in structural engineering.

However, advances in composite materials of high resilience capacities and low shear stiffness as well as the need for lighter

* Tel.: +57 42686218; fax: +57 44255152.
E-mail address: jdaristi2@yahoo.com.

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doi:10.1016/j.engstruct.2008.02.007

Nomenclature

A_s Effective shear area of the beam–column cross section;
E Young's modulus of the material;
G Shear modulus of the material;
L Length of the beam–column AB;
I Principal moment of inertia of the beam–column about its axis of bending;
M_a and M_b Bending moments (clockwise +) at ends A and B, respectively;
P Applied axial load at ends A and B (+ compression, − tension);
P_{cr} Critical axial load;
P_e $\pi^2 EI/L^2$ = Euler load;
R_a and R_b Stiffness indexes of the flexural connection at A and B, respectively;
$u(x)$ Lateral deflection of the beam–column center line;
$\beta = \frac{1}{1+P/(GA_s)}$ Shear reduction factor;
Δ Sway of end B with respect to end A;
κ_a and κ_b Flexural stiffness of the end connections at A and B, respectively;
ρ_a and ρ_b Fixity factors at A and B of column AB, respectively;
$\psi(x)$ Rotation of the cross section due to bending alone as shown by Fig. 1c;
$\psi_{a'}$ and $\psi_{b'}$ Bending rotations of cross sections at ends A′ and B′ with respect to cord A′B′, respectively;
$\phi = \sqrt{|P/(\beta EI/L^2)|}$ Stability function in the plane of bending;
θ_a and θ_b Rotations of ends A and B due to bending with respect to the vertical axis, respectively [$\theta_a = \psi_{a'} + \frac{M_a}{\kappa_a}$ and $\theta_b = \psi_{b'} + \frac{M_b}{\kappa_b}$];
$\Gamma = \frac{12(EI/L^2)}{GA_s}$ Bending to shear coefficient.

and stronger beams and columns have created a great interest in the shear effects and second-order analysis of framed structures. For instance, elastomeric seismic isolators and members made of light polymer materials may undergo extremely large deflections under combined axial and transverse loads without exceeding their elastic limit. The slope-deflection equations for Timoshenko beams including the effects of shear deformations and transverse loads were developed by Bryan and Baile [6]. Previously, Lin, Glauser and Johnson [7] had developed the slope-deflection equations for laced and battened beam–columns including the effects of shear deformations, axial load and end rigid stay plates. On the other hand, the stability and second-order analysis of slender beam–columns and of framed structures with semi-rigid connections has been investigated by Aristizabal-Ochoa [8,9] using the classical stability functions. However, the stability of framed structures using the classic slope-deflection method including the combined effects of shear and bending deformations, second-

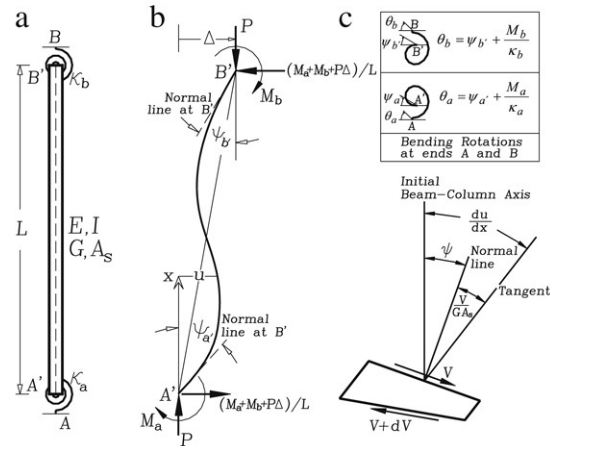

Fig. 1. Beam–column under end moments with semi-rigid bending connections at ends A and B: (a) Structural model; (b) Degrees of freedom, forces and moments in the plane of bending; (c) Rotations at a cross section and at ends A and B.

order P–Δ effects, and semi-rigid end connections is not known yet.

The main objective of this publication is to present a new set of slope-deflection equations for the stability and second-order analysis of framed structures made of Timoshenko beam–columns of symmetrical cross section with semi-rigid connections under end loads including the combined effects of: (1) bending and shear deformations; and (2) the shear component of the applied axial forces (Haringx's Model). The proposed method which is based on the "modified" stability functions for Timoshenko beam–columns with semi-rigid connections (Aristizabal-Ochoa [10,11]) has the following advantages: (1) the effects of semi-rigid connections are condensed into the slope-deflection equations for tension or compression axial loads without introducing additional degrees of freedom and equations; (2) it is more accurate than any other method available and capable of capturing the phenomena of buckling under axial tension forces of short columns like laminated elastomeric bearings commonly used for seismic isolation of buildings; and (3) the method is powerful, practical, versatile and easy to teach. Four comprehensive examples are included that show the effectiveness of the proposed method and corresponding equations.

2. Structural model

Assumptions. Consider a 2-D prismatic beam–column that connects points A and B as shown in Fig. 1a. The element is made up of the beam–column itself A′B′, and the flexural connections AA′ and BB′ with bending stiffness κ_a and κ_b at ends A and B, respectively. It is assumed that the beam–column A′B′ of span L bends about the principal axis of its cross section with a moment of inertia I, effective shear area A_s and: (1) is made of a homogeneous linear elastic material with Young and shear moduli E and G, respectively; (2) its centroidal axis is a straight line; and (3) is loaded at extremes A and B with P (compressive axial load is +) along its centroidal axis.

The flexural connections have stiffnesses κ_a and κ_b (whose units are in force–distance/radian) in the plane of bending of