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Nonlinear behavior of dynamic systems with high damping rubber devices

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ABSTRACT

The High Damping Rubber (HDR) is widely used in seismic engineering and, more generally, in the passive control of vibrations. Its constitutive behaviour is quite complex and is not simply non-linear with respect to strain but also shows a transient response during which material properties change (Mullins effect). A number of recent works were dedicated to analyzing and modelling the material behaviour. The present work intends to study the consequences of such non-linear behaviour in the dynamic response of S-DoF systems where the restoring force is provided by dissipative devices based on the HDR (structural system with dissipative bracings and isolated systems). Preliminary analyses under harmonic forces and impulsive excitations were carried out in order to separately characterize stable and transient responses. Finally, the response under seismic inputs with different intensities was studied. Results show that the Mullins effect may play an important role in the seismic response and the dynamic properties of the system change significantly for seismic events with different intensities.

1. Introduction

In the field of seismic engineering the rubber with enhanced dissipating properties, usually known as High Damping Rubber (HDR), is extensively adopted in bearings for the seismic isolation of bridges or buildings [1], and is also used for dissipating devices in order to increase stiffness and the energy dissipation capacity of structures [2–5]. With respect to other types of damper devices, based on elasto-plastic or viscous materials, the HDR-based damper seems to be a promising energy dissipating device because no permanent strains occur after seismic events and moreover it permits dissipating energy even for small lateral displacements produced by wind or minor earthquakes.

Some difficulties in the use of this kind of dissipating device derive from its complex dynamic behaviour which, makes it difficult to evaluate the behaviour of equipped structures accurately and to give design indications. More specifically, the material behaviour is strongly non-linear and both stiffness and damping properties vary with the amplitude of strain and depend on the strain rate [6,7]. Furthermore, the presence of filler added to the natural rubber, makes the response of the HDR strain history-dependent and causes a transient behaviour in which stiffness and damping change remarkably. The phenomenon, usually known as the "Mullins effect" or "scragging", is a consequence of the damage of the microstructure, that occurs during the process [8,9]. Recent studies [10] showed that the transient response is related to the maximum strain attained by the material and is influenced by the strain rate. The initial properties of the material may be however recovered (healing effect) and the healing times depend on the material considered and on the temperature [10]. The rubber studied in [11] showed a rapid recovery of a large part of the material stiffness even if the complete recovery may take several months. Consequently seismic analyses of structures endowed with HDR devices should be performed with virgin material properties, even if some scragging process were applied to the devices, and a model with damage should be adopted. The influence of scragging on the seismic response is also evidenced in [12] where the bidimensional response of an isolated bridge is analyzed.

The present work intends to analyze the dynamic response of single degree of freedom systems in which the restoring force is given by HDR devices, by using a unidimensional model based on virgin properties of the rubber and including the scragging phenomenon, previously developed by the authors [11]. The aim of these numerical analyses is to evidence characteristic aspects which can be of interest in the structural design under seismic actions and which cannot be described by simpler models, such as linear visco-elastic or elasto-plastic, usually used to simulate the HDR-based devices behaviour [13,14]. The analyses consider a range of the shear strain from 0.0 to 2.0, which seismic dissipating devices usually undergo. Three different ratios between mass and stiffness have been considered in order to study the rubber response in different dynamic situations, spanning from vibrations with long period, which furnish information about isolated systems, to vibrations with short period, which furnish

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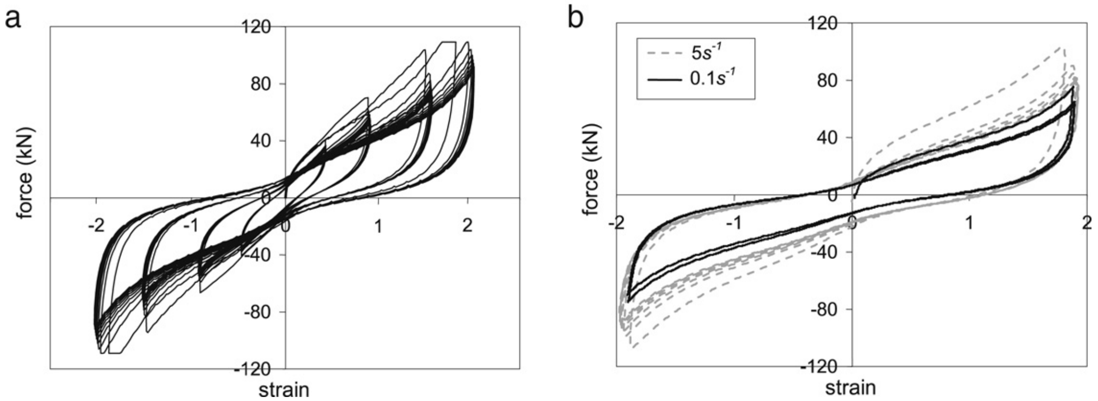

Fig. 1. Mullins effect: different strain amplitudes (a) different strain rate (b).

information about the rubber response in dissipating braces inserted within deformable frames.

In particular, Section 3 investigates the harmonic behaviour of the dynamical systems subjected to sinusoidal forces. The results refer to the stable, post-transient, response and furnish information regarding the influence of non-linear behaviour of the HDR on the dynamic response of the system, once the Mullins effect is over.

The following section is otherwise oriented to highlight the influence of the Mullins effect that influences the initial part of the dynamic response. For this purpose, the system behaviour under an impulsive initial input is studied.

Finally, the last section is dedicated to the analysis of the system subjected to seismic inputs. The analyses show that the Mullins effect may influence and change the system response in the case of similar seismic events (similar frequency content and maximum displacement attained). Furthermore, the influence of all the non-linear phenomena on the response under seismic events of different intensities is analyzed in order to show how the dynamic properties of the system change by varying the input intensity.

2. Dynamical system

2.1. HDR model

The model used to simulate the HDR response is a rheological unidimensional model able to describe the transient response of the rubber, depending on both the strain rate and the maximum strain attained, as evidenced by experimental tests reported by Fig. 1. In the model the *state* of the material is furnished by the shear strain γ, defined as the ratio between the shear displacement and the thickness of the rubber, and by a set of internal variables α_i describing the inelastic response and the Mullins effect. The tangential stress may be derived from the free energy per unit volume $\varphi_d\left(\gamma;\alpha_i\right)$ by the relation

$$\tau_d = \frac{\partial \varphi_d}{\partial \gamma} \tag{1}$$

whereas the dissipated power per unit volume w_d may be obtained from the derivative with respect to the internal variables (repeated indexes denote summation, superposed dot denotes time derivative)

$$w_d = \frac{\partial \varphi_d}{\partial \alpha_i}\dot{\alpha}_i. \tag{2}$$

The stress deriving from a strain history may be determined once the initial state and the *process* $\eta = \dot{\gamma}$ are known, on the basis of the nonlinear functions $g_i\left(\gamma,\eta;\alpha_i\right)$, which describe the evolution of the internal variables:

$$\begin{bmatrix}\dot{\gamma}\\ \dot{\alpha}_i\end{bmatrix} = \begin{bmatrix}\eta\\ g_i\left(\gamma,\eta;\alpha_i\right)\end{bmatrix}. \tag{3}$$

The expressions of the free energy and those of the evolution functions adopted in the following analyses are reported and commented in the Appendix.

2.2. S-DoF system

The S-DoF (Single-Degree of Freedom) dynamical system considered consists of a mass m and an HDR-based dissipating device that furnishes the restoring force. It is assumed that a linear relation, defined by a constant c, exists between the mass displacement u and the shear strain of the device rubber $\gamma = cu/h$, where h is the thickness of the rubber layer. The value of c depends on the geometry of the connection between the device and mass. The restoring force per unit mass f_d can be expressed in the form

$$f_d = \frac{cA}{m}\tau_d \tag{4}$$

where A is the area of the *HDR* layer in the device. The *state* of the system is consequently described by the vector $\mathbf{x} = \left[u, v; \alpha_i\right]$ where v is the velocity of the mass. The evolution law has the following form:

$$\dot{\mathbf{x}} = \begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{\alpha}_i\end{bmatrix} = \begin{bmatrix} v\\ -f_d\left(c\dfrac{u}{h}, c\dfrac{v}{h}; \alpha_i\right) + f_e\\ g_i\left(c\dfrac{u}{h}, c\dfrac{v}{h}; \alpha_i\right)\end{bmatrix} = \mathbf{A}\left(\mathbf{x}\right) \tag{5}$$

where f_e is the external force per unit mass. It may be useful to observe that the following non-linear results may also be extended to cases different from those considered here. The equation of motion can be rewritten by remembering that $u = \gamma h/c$ and by dividing each term by the thickness h. The equation assumes the form

$$\dot{\mathbf{x}} = \begin{bmatrix}\dot{\gamma}\\ \ddot{\gamma}\\ \dot{\alpha}_i\end{bmatrix} = \begin{bmatrix}\gamma\\ -\dfrac{cA}{mh}\tau_d\left(\gamma,\eta;\alpha_i\right) + \dfrac{f_e}{h}\\ g_i\left(\gamma,\eta;\alpha_i\right)\end{bmatrix} \tag{6}$$

consequently the same strain history may be observed in all those cases where the two ratios cA/mh and f_e/h did not vary (*e.g.* non-linear response does not vary by doubling f_e if both A and h are also doubled).