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Linear buckling analysis of unstiffened plates subjected to both patch load and bending moment

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ABSTRACT

In this study, linear buckling analysis of unstiffened plates under interacting patch loading and bending moment is developed. The study focuses on estimation of the elastic critical load due to patch load, and concomitant linearly variable compressive stress in the normal direction, with analysis of mechanisms of plate instability.

The present work proposes simple design equations for the elastic critical load of rectangular plates subjected to both patch load, and uniform compressive stress, and also patch load with linearly varying compressive stresses for the serviceability limit state. The proposed analytical relationship, obtained on the basis of parametric numerical analyses, is validated by comparing analytical predictions with experimental tests from the literature, and full numerical models; good agreement was obtained for practical situations in steel bridge design, such as the erection of steel bridges with the launching construction technique. In the launch phase, web panels of the open-section girders are subjected to concentrated loads applied to the lower flange, and considerable bending moment.

1. Introduction

Steel girder web panels subjected to interacting patch loading and bending moment at the serviceability limit state has not been deeply studied in the literature. However, this is a very common load situation for incremental launching of bridge girders, in which girders are subjected to concentrated loads applied to the inferior flange and to considerable bending moment. The aim of this paper is to propose a simple design formulation for the elastic critical load in steel web panels subjected to patch load, and concomitant linearly variable compressive stress in the normal direction for the serviceability limit state.

A typical welded double-T cross-section is considered, subjected to a uniformly distributed load along length l_o on the upper flange and to bending moment M_z acting in the plane of the plate. The generic panel has length (between two ribs) a, height h and thickness t. Linear-elastic and isotropic steel with Young's modulus $E = 206\,000$ N/mm^2 and Poisson's ratio $\nu = 0.3$ is examined.

The common conservative assumption of neglecting the contribution of the upper and lower flanges, and the vertical stiffeners of the beam in determining of the linear buckling load of an open-section girder is adopted (Lagerqvist and Johansson 1996 [1]). The boundary conditions are as follows (Fig. 1): z-displacement is restrained for the four edges (1, 2, 3, 4) and y-displacement is also restrained for the vertical edges (2, 4). The plate is subjected to vertical concentrated load F_y applied at edge 1 and bending moment M_z acting at edges 2 and 4. A plate without out-of-plane imperfections is considered, and the elastic critical load corresponds to the first linear buckling mode (Massonnet and Janss 1981 [2]). The topic of linear stability analysis of simply supported rectangular plates is amply discussed in the literature (Shahabian and Roberts 1999 [3], Ren and Tong 2005 [4]), but the common design situation of plates subjected to both patch load and bending moment has not been deeply studied and, to the authors' knowledge, there are no simple design relationships which estimate the elastic critical load for this situation.

The results of this work may be used for design purposes in practical applications in steel bridge construction, particularly for the incremental launching technique.

2. Linear buckling analysis

Finite Element Code Straus [5] is used in the following buckling analyses. It solves linear buckling problems, by finding the buckling load through the solution of the eigenvalue problem. The lower eigenvalue corresponds to the elastic critical load, and the eigenvector defines the corresponding deformed shape.

Stiffness matrix $\boldsymbol{K}$ is formed from the conventional matrix of small deformations, and has constant value $\boldsymbol{K}_E$ and matrix $\boldsymbol{K}_s$, which takes into account the effect of stress σ on the plate. The global stiffness matrix of the panel at stress level σ_0 may be written as follows:

$$\boldsymbol{K}(\sigma_0) = \boldsymbol{K}_E + \boldsymbol{K}_s(\sigma_0). \quad (1)$$

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Notation

a	plate width;
c_1	coefficient depending on h/a;
c_2	coefficient depending on l_o/a;
c_3	coefficient depending on F_x;
D	flexural rigidity of plate;
E	Young's modulus;
F_x	horizontal load applied to vertical edges;
$F_{cr,x}$	first linear buckling load in x direction;
F_y	patch load applied in y direction;
$F_{cr,y}$	first linear buckling load in y direction;
f_{yd}	design yield strength;
h	plate height;
k	linear buckling coefficient;
$k_{\sigma,p}$	buckling coefficient following plate stability theory;
l_o	patch load length;
M_z	bending moment acting around z axis;
t	plate thickness;
V	shear force;
α	panel aspect ratio ($=a/h$);
λ	plate slenderness;
λ_1	first eigenvector
ψ	stress ratio;
$\sigma_{cr,p}$	elastic critical stress;
σ_E	Eulerian critical stress;
$\sigma_{\sup}$	stress acting on upper edge;
$\sigma_{\inf}$	stress acting on lower edge;
σ_x	stress acting in x direction;
σ_0	compressive stress;
ν	Poisson's ratio.

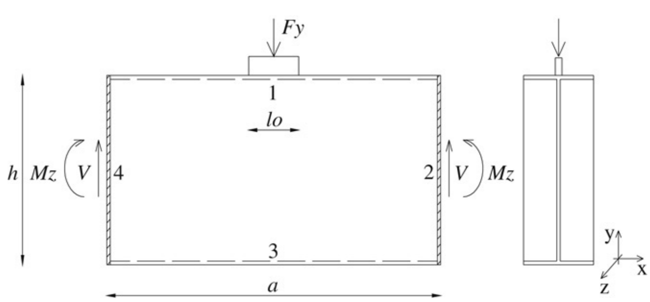

Fig. 1. Plate geometry with load scheme and girder cross-section.

When the stress level reaches $\lambda\sigma_0$, the stiffness matrix becomes:

$$\boldsymbol{K}(\lambda\sigma_0) = \boldsymbol{K}_E + \boldsymbol{K}_s(\lambda\sigma_0) = \boldsymbol{K}_E + \lambda\boldsymbol{K}_s(\sigma_0). \tag{2}$$

The equation that governs the behaviour of the plate is:

$$d\boldsymbol{F} = [\boldsymbol{K}_E + \lambda\boldsymbol{K}_s(\sigma_0)]d\boldsymbol{u} \tag{3}$$

where $d\boldsymbol{u}$ is the vector of increasing displacement and $d\boldsymbol{F}$ the vector of increasing load. The determinant of the matrix becomes null at buckling, and an increase in displacement without a corresponding increase in load occurs:

$$[\boldsymbol{K}_E + \lambda\boldsymbol{K}_s(\sigma_0)]d\boldsymbol{u} = 0. \tag{4}$$

This is an eigenvalue problem, the solution to which corresponds to the lower eigenvalue λ_1 related to critical elastic stress, at which buckling occurs:

$$\sigma_{cr} = \lambda_1\sigma_0. \tag{5}$$

3. Plate under patch load action only

According to the classical theory of plate stability, the elastic critical load of a simply supported plate is generally calculated through the following well-known equation (see, for example, Chatterjee 2003 [6]):

$$F_{cr,y} = k\frac{\pi^2 E}{12(1-\nu^2)}\frac{t^3}{h}. \tag{6}$$

The term $D = Et^3/12(1-\nu^2)$ is the flexural stiffness of the plate. The elastic critical load mainly depends on the geometry of the plate (thickness t and height h) that defines slenderness λ. Formulas to calculate the buckling coefficient for varying restraints and load conditions are proposed in a number of papers. Lagerqvist and Johansson (1996) [1] proposed empirical equations for the buckling coefficient, also taking into account the influence of flange geometry.

Load length is an important parameter for buckling analysis in patch load conditions: the influence of load length is shown in Fig. 2, where the same plate with $a/h >> 1$, and subjected to loads with different lengths l_o, shows different instability configurations. A plate with $a = 4500$ mm, $h = 650$ mm, $t = 16$ mm ($\alpha = a/h = 6.9$ and $\lambda = h/t = 41$), $l_o = 1200$ mm (Fig. 2(a)) and $l_o = 350$ mm (Fig. 2(b)) was studied with FE code Straus7 [5], using plate elements with four nodes (Quad4), and six degrees of freedom for each node. The four plate edges were simply supported in the out-of-plane direction, and vertical translation was restrained at the lateral edges to prevent rigid body movements.

Fig. 2(a) and (b) show that instability mainly occurs in the upper part of the plate, with more than one half-wave (three in Fig. 2(a)) for large load lengths l_o with respect to plate length a, whereas instability involves the whole plate height with one half-wave (Fig. 2(b)) for short load lengths l_o.

Duchene and Maquoi (1994) [7] proposed some formulas to calculate linear buckling coefficient k in the following ranges of panel aspect ratio: $0.5 \le \alpha \le 1.8$, $1.8 \le \alpha \le 4$ and $\alpha \ge 4$. According to an extensive linear buckling parametric numerical analysis performed with FE code Straus7 [5], with plate geometry and load length as parameters, the following formula is proposed here for buckling coefficient k:

$$k = 3.2048\frac{c_1}{c_2}. \tag{7}$$

c_1 and c_2 may be obtained with two polynomial relationships, which take into account the influence of the ratio between plate height h and plate width a and the ratio between the length of load l_o and plate width a respectively:

$$c_1 = -0.0446\left(\frac{h}{a}\right)^6 + 0.4067\left(\frac{h}{a}\right)^5 - 1.3512\left(\frac{h}{a}\right)^4 + 1.8338\left(\frac{h}{a}\right)^3 - 0.3869\left(\frac{h}{a}\right)^2 - 0.2617\left(\frac{h}{a}\right) + 0.8035 \tag{8a}$$

$$c_2 = 1.9774\left(\frac{l_o}{a}\right)^6 - 6.0815\left(\frac{l_o}{a}\right)^5 + 7.1163\left(\frac{l_o}{a}\right)^4 - 3.5774\left(\frac{l_o}{a}\right)^3 + 0.2617\left(\frac{l_o}{a}\right)^2 - 0.1669\left(\frac{l_o}{a}\right) + 1 \tag{8b}$$

c_1 and c_2, may be also obtained with a lower degree of approximation, with simpler polynomial equations:

$$c_1 = -0.0813\left(\frac{h}{a}\right)^3 + 0.512\left(\frac{h}{a}\right)^2 - 0.0398\left(\frac{h}{a}\right) + 0.6167 \tag{8c}$$

$$c_2 = 0.3466\left(\frac{l_o}{a}\right)^3 - 0.7462\left(\frac{l_o}{a}\right)^2 - 0.0699\left(\frac{l_o}{a}\right) + 0.999. \tag{8d}$$