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Deformability of concrete beams with unbonded FRP tendons

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ABSTRACT

As fibre-reinforced polymer (FRP) behaves linearly until it ruptures without any yielding, the ductility and deformability of prestressed concrete beams with FRP tendons are of prime concern. This paper describes a comparative study of five existing deformability indices for unbonded partially prestressed concrete (UPPC) beams with FRP tendons. A numerical method has been developed to predict the full-range response of prestressed concrete beams with bonded and/or unbonded FRP tendons under loading, and the results agree well with experimental results reported in the technical literature. The results show that the deformability index defined as the ratio of the product of moment and deflection at ultimate to the corresponding value at cracking is consistently decreasing with the increase of combined reinforcement ratio. In addition, this index is sensitive to changes of factors that may influence the deformability of UPPC beams with FRP tendons. The present investigation also indicates that, when the ratio of neutral axis depth to the depth to FRP tendons at the critical section at ultimate is greater than 0.3, the beam would fail by crushing of concrete, which is the more favourable failure mode compared to the rupture of tendons.

1. Introduction

Fibre reinforced polymers (FRP) usually present high strength, light weight, corrosion resistance, and insensitivity to magnetic effects. Unlike steel, FRP materials are brittle in nature with linearly elastic behaviour until failure. Therefore the ductility or deformability is one of the main concerns in prestressed concrete beams with bonded and/or unbonded FRP tendons. For steel reinforced or prestressed concrete members, ductility is often quantified by the ratio of values of displacements or curvatures at ultimate failure to that at the yielding of steel. Because FRP materials do not yield, the term deformability was introduced by researchers as a means of assessment of the displacement or curvature that occurs before rupture of the FRP tendons. Researchers have proposed several different models to evaluate the deformability of prestressed concrete beams with FRP tendons, but there is still lack of general agreement as to how the deformability characteristics of such members may be quantified and analysed.

It is well known that the ultimate strain of FRP tendons is lower than that of high strength steel and that the strain in unbonded prestressing tendons at the critical section of a beam is smaller than that in bonded prestressing tendons under the same loading. Therefore Burgoyne [1] advocated that for concrete beams prestressed with FRP tendons, the tendons should be unbonded to concrete. This argument has been proved by experiments as well as practical bridge projects [2,3].

This paper describes a comparative study of existing deformability models for FRP prestressed concrete beams suggested by different investigators in recent years. A numerical method has been developed to predict the full-range response of prestressed concrete beams with bonded and/or unbonded FRP tendons under loading, and the results agree well with experimental results reported in the technical literature. A parametric study to evaluate the trend of different proposed models for prediction of the deformability in unbonded partially prestressed concrete beams with FRP tendons will be presented.

2. Review of existing deformability indices

2.1. Energy-based models

Naaman and Jeong [4] first introduced an energy-based deformability index (the Naaman Index) given as

$$\mu_{en} = 0.5\left(\frac{E_{tot}}{E_{el}} + 1\right) \quad (1)$$

where E_{tot} is the total energy, computed as the area under the

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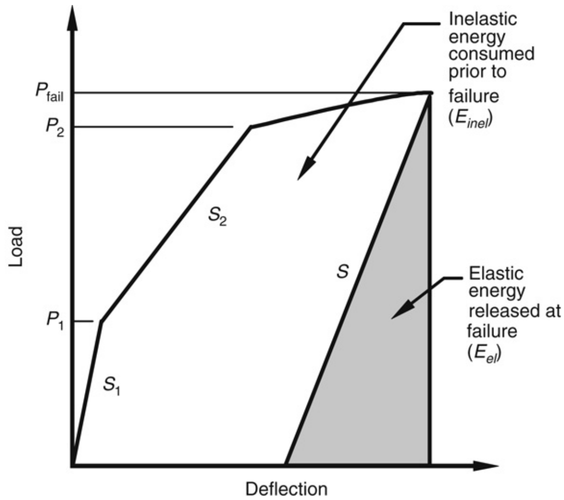

Fig. 1. Total, elastic and inelastic energy.

load deflection curve up to the load defined as the failure load, and E_{el} is the energy, which is a part of the total energy E_{tot}, as shown in Fig. 1. Naaman and Jeong [4] suggested that the elastic energy could be estimated using an equivalent triangular area under the load–deflection curve. The two initial slopes S_1 and S_2, corresponding to applied loads P_1 and P_2, of the load–deflection curve as illustrated in Fig. 1 are weighted to define a slope S for the equivalent unloading response as

$$S = \frac{P_1 S_1 + (P_2 - P_1) S_2}{P_2}. \quad (2)$$

Similar to the model of Naaman and Jeong, Grace et al. [5] used the ratio of inelastic energy to total energy to quantify the deformability of FRP-reinforced beams (the Grace Index). According to Grace et al. [5], if the energy ratio is 75% or greater, then the beam will exhibit a ductile failure. As pointed out by Tann et al. [6], the Grace model is actually a reversed form of the Naaman model. If the energy ratio of Grace et al. is 75%, then E_{tot}/E_{el} would be $1/(1-0.75) = 4$, and the deformability index as determined by Eq. (1) would be 2.5, which is naturally an indication of acceptable ductility level.

The above energy-based deformability index is dependent on how much of the total energy is elastic and how much is inelastic. The index is also affected by the load level at which unloading begins [7]. In order to account for the influence of Young's modulus and failure strength of the reinforcement, type of reinforcing bars and stirrups, failure mode of the beam, and concrete softening at compressive flexural failure on the slope S, Grace et al. [5] further introduced 4 parameters to Eq. (2).

Saafi and Toutanji [8] later put forward another energy-based deformability index defined as the ratio of the entire area under the load–deflection curve to the area under the load–deflection response up to the deflection corresponding to the maximum load. In this method, the post-peak response of the beam should be known first but it is not always available.

ACI 440 [9] introduced a deformability index as the ratio of energy absorption (area under the moment-curvature curve) at ultimate strength of the section to the energy absorption at service level. Concerning the service level, Vijay and GangaRao [10] subsequently suggested a limiting curvature value based on the serviceability criteria of both deflection and crack width.

2.2. Deflection-based models

Abdelrahman et al. [11] established a deformability model based on deflection for beams prestressed by FRP tendons.

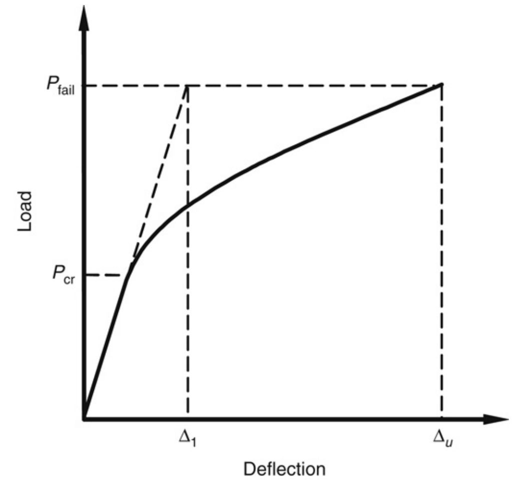

Fig. 2. Equivalent deflection Δ_1 and failure deflection Δ_u.

This Abdelrahman deformability index (the Abdelrahman Index) A_b is expressed as the ratio of the maximum deflection Δ_u corresponding to the failure or maximum load to the equivalent deflection Δ_1 of the uncracked section at a load equal to the ultimate load (Fig. 2), namely

$$A_b = \frac{\Delta_u}{\Delta_1}. \quad (3)$$

According to Abdelrahman et al. [11], this definition overestimates ductility for prestressed concrete beams with steel. This method represents the bilinear elastic deformation of beams prestressed by FRP and is different from the traditional ductility evaluation in terms of the inelastic deformation of the system prior to collapse.

Rashid et al. [12] experimentally studied the behaviour of FRP-reinforced high strength concrete beams, with the concrete in the compression zone confined. It was found that the load–deflection response of such beams showed a first peak at the initiation of cover concrete crushing and a second peak, usually higher than the first one, before the confined concrete in the compression zone finally disintegrated. Therefore they proposed the deflection-based deformability index (the Rashid index) R as

$$R = \frac{\Delta_u}{\Delta_{crush}} \quad (4)$$

where Δ_u is the maximum deflection corresponding to the failure or maximum load and Δ_{crush} is the deflection at the initiation of cover concrete crushing.

2.3. Moment and deformation-based models

Jaeger et al. [13] and the Canadian Highway Bridge Design Code [14] used a deformability index (the Jaeger Index) taking into account moment and curvature at ultimate state, as well as the moment and curvature at service limit state. The Jaeger Index J is defined as

$$J = \left(\frac{\varphi_u}{\varphi_{0.001}}\right)\left(\frac{M_u}{M_{0.001}}\right) \quad (5)$$

where M_u and φ_u are the moment and curvature at ultimate state, respectively; $M_{0.001}$ and $\varphi_{0.001}$ are the moment and curvature at service limit state, respectively. The service limit state corresponds to a concrete strain at the top compression fibre of 0.001. The Canadian Highway Bridge Design Code [14] requires the