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Effect of inclination on bending of variable-arc-length beams subjected to uniform self-weight

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Abstract

This paper presents the effect of inclination on the static behaviors of inclined variable-arc-length (VAL) beams. An updated Lagrangian formulation of the VAL beams has been developed via the variational approach. The first variation of the total potential energy is evaluated to establish the system of nonlinear finite element equations. These equations are then solved by using the iterative process to obtain static configurations. The second variation of the total potential energy is performed to determine the tangent stiffness matrix of the VAL beams. The critical values of uniform self-weight of the inclined VAL beams are obtained by equating the determinant of the tangent stiffness to zero. The critical uniform self-weights obtained from the finite element method are validated by those derived from the shooting method. The effects of inclination on the deflection, bending moment diagram, and axial force diagram at critical state are investigated herein.

Keywords: Variable-arc-length beams; Inclined beams; Finite element method; Critical self-weight; Large deflection; Nonlinear; Shooting method

1. Introduction

Variable-arc-length (VAL) beam is a class of beam of which an end is a hinged support and the other end at a fixed distance is propped by a frictionless roller over which the beam can slide freely. In previous studies, the buckling of horizontal VAL beams subjected to end moments and/or point loads were presented by using elliptic integrals [1–5] in order to attain the exact solutions. Nevertheless, the solutions of VAL beams subjected to uniform self-weight cannot be obtained by the elliptic integral procedure. Therefore, the finite element method (FEM) and the shooting method (SM) are more efficient for use in order to take care of this problem. The numerical results for large deflection of horizontal VAL beams subjected to uniform self-weight, which were obtained from FEM and SM, were previously validated by the experimental examples [6].

From the literature review, the static behaviors for inclined VAL beam subjected to uniform self-weight have not yet been reported elsewhere. The major objective of this paper is thus to evaluate the buckling behaviors of inclined VAL beams subjected to uniform self-weight. Effects of inclination on the static behaviors of VAL beams are separately presented in two cases: shift-up and shift-down. In each case, the values of critical uniform self-weight of the inclined VAL beams with various inclinations are determined by the finite element method. These results are then independently checked by the shooting method.

The following assumptions are made throughout this analysis:

(1) An axial movement is unrestrained at the frictionless roller; therefore, the effect of axial deformation is not included in this analysis.
(2) The beam material is assumed to be homogeneous, isotropic, and linearly elastic.
(3) A cross-section of the beam remains plane and remains perpendicular to the axis.
(4) The effect of shear and torsional rigidities is neglected.

In practice, the inclined VAL beams are often considered for uses in either static or dynamic analyses of a flexible pipe/riser in offshore engineering applications where the flexible pipe/riser is generally used as a linkage between the floating platform and the transport vessel. The recent findings

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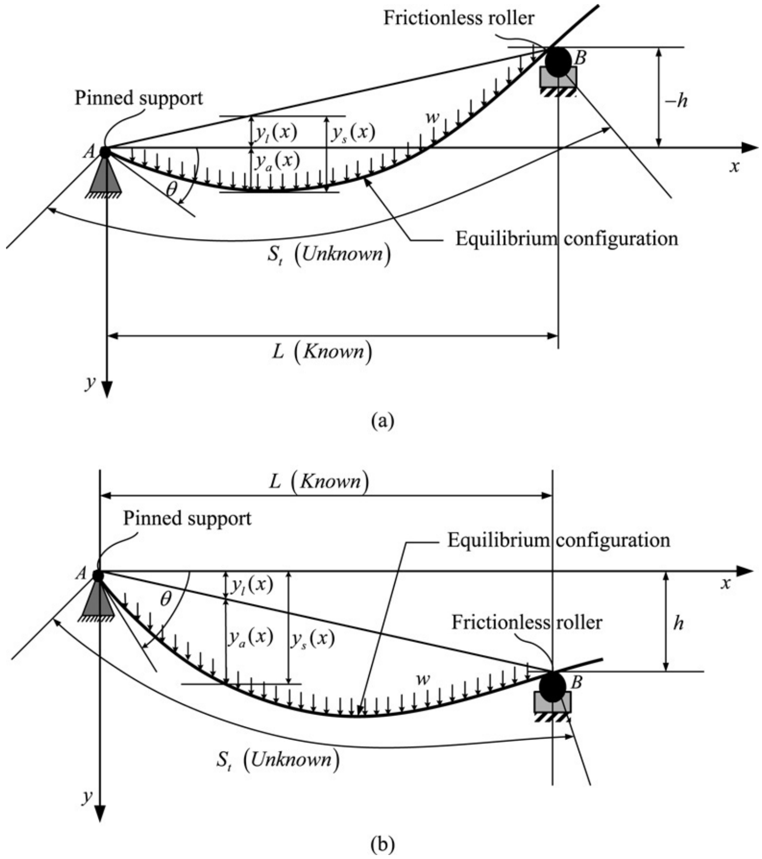

Fig. 1. (a) Equilibrium configuration of shift-up inclined VAL beam, (b) Equilibrium configuration of shift-down inclined VAL beam.

from this study will benefit the analysis and design of the flexible marine pipes/risers, as well as serve as the benchmarks for the future experimental investigations.

2. Finite element method

2.1. The updated Lagrangian formulation

Fig. 1(a) and (b) show static configurations of an inclined VAL beam. The beam is supported by a pin at End A and by a frictionless roller at End B. End B locates at a fixed distance L from End A with the different level h. The span length L is a known constant whilst the total arc-length at equilibrium state S_t is an unknown parameter. Due to the uniform self-weight of beam w (weight per unit arc-length), the beam deflects with the total deflection y_s which is decomposed into linear and nonlinear parts, y_l and y_a respectively.

The first variation of the total potential energy of the beam in the updated Lagrangian Cartesian coordinates [7] is

$$\delta\pi_s = \int_0^L \left\{ \frac{EIy_s''}{\left(1+y_s'^2\right)^{5/2}}\delta y_s'' - \delta y_s' \frac{2EIy_s''^2 y_s'}{\left(1+y_s'^2\right)^{7/2}} + \delta y_s' \frac{Ny_s'}{\sqrt{1+y_s'^2}} - \delta y_s w\sqrt{1+y_s'^2} \right\} \mathrm{d}x. \quad (1)$$

The first and second terms in Eq. (1) are the consequence of bending energy. The third term is attributed to the axial force. The last term is the resultant of the uniform self-weight of the beam. The superscript ($'$) represents the derivatives with respect to x. The nomenclatures E, I, and N in Eq. (1) are the elastic modulus of the beam, the moment of inertia of the beam section, and the axial tension, respectively.

2.2. Solution procedure for large displacement analysis

At an equilibrium state, $\delta\pi_s = 0$ is applied. Because the total arc-length of the beam is not given earlier, the use of arc-length as the independent variable, as commonly used in conventional beam element, may not be convenient for establishing the boundary conditions. However, in this case the span length is known while the total arc-length has to be determined. Therefore, the discretization of span length is used instead of the total unknown arc-length. By using this technique, the boundary conditions can be conveniently established. Moreover, the discretization of span length yields span elements with known element length instead of beam elements with unknown element length (see Fig. 2). The linear part y_l can be directly obtained from the linear proportion with the specified coordinate x, while the nonlinear part y_a is approximated by the fifth order polynomial, thus

$$y_s(x) = y_a(x) + y_l(x), \qquad y_a(x) = \lfloor N \rfloor \{q_s\} \quad (2)$$

in which $\lfloor N \rfloor$ is the row matrix of the fifth order shape functions; and $\{q_s\}$ is the corresponding local degrees of