Available online at www.sciencedirect.com

ScienceDirect

Engineering Structures 30 (2008) 990–1001

ENGINEERING STRUCTURES

www.elsevier.com/locate/engstruct

Efficient analysis of structural uncertainty using perturbation techniques

A.A. Henriques*

Faculty of Engineering of the University of Porto, Department of Civil Engineering, Rua Dr. Roberto Frias, 4200-465, Porto, Portugal

Received 16 January 2006; received in revised form 20 June 2007; accepted 21 June 2007
Available online 9 August 2007

Abstract

Probabilistic and reliability techniques have been used increasingly to evaluate uncertainty and structural safety. However, when sophisticated methods of structural analysis are used, the current probabilistic techniques require the execution of various simulations for the same problem. This has been one of the main factors to restrain the use of this type of techniques. In this work, the formulation of an efficient method to evaluate the uncertainty of the structural response by applying perturbation techniques is described. The procedure used to implement this method in a structural non-linear finite element framework is presented. The implemented computational program allows, in only one structural analysis, to evaluate the mean value and the standard deviation of the structural response, defined in terms of displacements or forces. The results are exact for linear problems and normal distributed random variables. Calculated values remain accurate when the non-linear problem can be approximated by a linear combination of the basic random variables more correlated with the structural response.

Keywords: Structural analysis; Uncertainty; Reliability; Perturbation; Non-linear finite elements

1. Introduction

The last years witnessed an increasing application of probabilistic techniques on the analysis of structural engineering problems, but until now the generalized application has been delayed by the inefficiency of these methods to solve complex or large problems. Nowadays, structural reliability concepts are widely acceptable and their application is rather simple when an explicit formulation of the structural problem exists. However, when there are no explicit relations between variables, such as in the finite element method, usually several analysis of the same problem should be performed to evaluate the uncertainty of structural response.

The first published papers dealing with reliability techniques and the finite element method were restricted to linear structural behaviour or to simplified non-linear behaviour [1–4]. Later works have presented proposals and applications of reliability techniques to the non-linear methods of structural analysis [5–9]. Recently, several methodologies and models have been presented. They take into account the uncertainty of parameters on complex structural models [10–16]. These methods can broadly be divided into three categories: reliability methods, perturbation methods and simulation methods.

In reliability methods the main purpose is to evaluate the probability of failure, by dividing the structural uncertainty space into safe domain and failure domain. The principles for incorporating the reliability methods in non-linear finite element frameworks have been presented by Liu and Der Kiureghian [6], using first-order and second-order reliability methods (currently named FORM and SORM).

Perturbation methods involve first- and second-order Taylor series expansion of the governing equations. The structural behaviour is characterized by taking into account, the terms around the mean values of the basic random variables. Mean and variance of the response can be found in terms of mean and variance of the basic random variables, thus, distribution information is not required [8,9,17,18].

Among simulation methods, the most straightforward technique is the Monte Carlo method [19], however, it requires a large number of random samples to evaluate the very small probabilities common in structural safety problems. Alternative techniques are usually used to reduce the sample size. One of these techniques is the response surface method [7,12,20,21] which combines the reliability techniques with simulation. The

* Tel.: +351 22 5081862; fax: +351 22 5081835.
E-mail address: abel.henriques@fe.up.pt.

0141-0296/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2007.06.007

latin hypercubic method is a technique to reduce the variance of estimation and is also widely used [22,23,13].

In previous works, the author presented methodologies to evaluate the structural safety based on the application of different reliability techniques, especially applied to non-linear analysis of concrete structures [24–26]. For these cases, the safety analysis to ultimate limit states can involve situations where the structural response has a non-normal distribution. In these circumstances, the evaluation of structural reliability demands a considerable number of simulations. However, generally the structural responses have normal or quasi-normal distributions. When this happens, it is possible to evaluate more efficiently the structural reliability.

In the present work, an efficient method to evaluate the uncertainty of structural response by applying perturbation techniques is presented. The variables of the structural problem with random nature are described by their mean values, standard deviation and correlation coefficients that quantify the dependence between these variables. The present procedure evaluates the mean response and its standard deviation in only one structural analysis. The results obtained are accurate when the normal distribution of structural response is guaranteed. The necessary procedures to implement these techniques in a finite element framework are described.

2. Formulation of the method

Structural system behaviour depends on different material parameters, elements' geometry and applied actions. Random nature of structural response is a consequence of structural parameters variability. Their quantification can be made by direct relationships between the dispersion of structural response and the dispersion of different parameters involved on the structural design.

Let the structural system be subjected to a loading level defined by: $F \cdot \Phi = F \cdot [\Phi_1, \Phi_2, \ldots, \Phi_n]$; where F is the load value and $[\Phi_1, \Phi_2, \ldots, \Phi_n]$ is the load distribution along the structure. In a load-test, the load value F increases successively until the studied limit state is achieved, but the load distribution (Φ vector) remains constant. When the finite element method is applied, the system equilibrium is defined by the following equation:

$$K(u) \cdot U = F \cdot \Phi \tag{1}$$

where $K(u)$ is the stiffness matrix, defined as a function of nodal displacements U, $F \cdot \Phi$ stands for the nodal force vector that represents the external actions.

When a perturbation δ is applied, the Eq. (1) becomes:

$$(K_0 + \delta K) \cdot (U_0 + \delta U) = (F_0 + \delta F) \cdot \Phi \tag{2}$$

in which variables with indexes 0 represent the central values of their probabilistic distribution (generally the mean values) and variables with δ-sign stand for perturbations around those central values.

Developing Eq. (2), and taking into account that $K_0 \cdot U_0 = F_0 \cdot \Phi$ and neglecting the second-order term, the following expression is obtained:

$$K_0 \cdot \delta U + \delta K \cdot U_0 = \delta F \cdot \Phi. \tag{3}$$

In this manner, the dispersion of structural response, in terms of displacements, can be evaluated by the relation:

$$\delta U = -K_0^{-1} \cdot \delta K \cdot U_0 + K_0^{-1} \cdot \delta F \cdot \Phi. \tag{4}$$

The non-deterministic nature of the structural design parameters is defined by random variables denoted by X. The covariance matrix of displacements, C_u, is calculated by:

$$C_u = \frac{\partial U}{\partial X} \cdot C_x \cdot \left(\frac{\partial U}{\partial X}\right)^{\mathrm{T}} \tag{5}$$

where C_x is the covariance matrix of random variables X, $\partial U/\partial X$ stands for the partial derivatives of displacements U with respect to the random variables X. The covariance matrix C_x is expressed by the standard deviation δX_i and the correlations ρ_{ij} between the m random variables X_i and X_j (where $i, j = 1, 2, \ldots, m$):

$$C_x = \delta X \cdot C_\rho \cdot \delta X^{\mathrm{T}} \tag{6}$$

in which C_ρ is the correlation matrix of random variables X and δX is a matrix containing the standard deviation of random variables X. Taking into consideration that the deviation of structural stiffness and the deviation of forces, defined in Eq. (4), result from the dispersion of random variables X, the deviation of structural response, in terms of displacements, can be defined by:

$$\begin{aligned} \delta U &= \frac{\partial U}{\partial X} \cdot \delta X \\ &= -K_0^{-1} \cdot \frac{\partial K}{\partial X} \cdot U_0 \cdot \delta X + K_0^{-1} \cdot \frac{\partial F}{\partial X} \cdot \Phi \cdot \delta X. \end{aligned} \tag{7}$$

Considering Eqs. (6) and (7), the covariance matrix of displacements, C_u, defined in Eq. (5), is computed as:

$$\begin{aligned} C_u = &\left(-K_0^{-1} \cdot \frac{\partial K}{\partial X} \cdot U_0 \cdot \delta X + K_0^{-1} \cdot \frac{\partial F}{\partial X} \cdot \Phi \cdot \delta X\right) \cdot C_\rho \\ &\cdot \left(-K_0^{-1} \cdot \frac{\partial K}{\partial X} \cdot U_0 \cdot \delta X + K_0^{-1} \cdot \frac{\partial F}{\partial X} \cdot \Phi \cdot \delta X\right)^{\mathrm{T}}. \end{aligned} \tag{8}$$

As follows, the dispersion of structural response, defined by matrix C_u, is evaluated by taking into account the mean values X_i, standard deviation δX_i and the correlations $\rho_{ij} (i, j = 1, 2, \ldots, m)$.

The dispersion of structural response can also be defined in terms of forces. This dispersion should be evaluated at the i-point where the displacement is maximum and the following condition is set: $\delta u_{i,\max} = 0$. Hence, Eq. (3) can be expressed by:

$$\delta K \cdot U_0 = \delta F_{i,\max} \cdot \Phi - K_0 \cdot \delta U|_{\delta U_{i,\max}=0} \tag{9}$$

or by the abridged equation:

$$\delta K \cdot U_0 = -K_M \cdot \delta q \tag{10}$$