Available online at www.sciencedirect.com
ScienceDirect
Engineering Structures 30 (2008) 1346–1359
ENGINEERING STRUCTURES
www.elsevier.com/locate/engstruct

A generalized analytical approach to the buckling of simply-supported rectangular plates under arbitrary loads

Y.G. Liu, M.N. Pavlović*

Department of Civil Engineering, Imperial College, London SW7 2BU, UK

Received 12 May 2007; received in revised form 14 July 2007; accepted 16 July 2007
Available online 7 September 2007

Abstract

An analytical approach for the elastic stability of simply-supported rectangular plates under arbitrary external loads is presented which, for the first time, may be described as 'exact'. This is achieved through the use of exact solutions for the in-plane stresses and the adoption of double Fourier series for the buckled profiles which, together, ensure that accurate results are obtained in the Ritz energy technique. Several cases of plate buckling under direct, shear and bending loads (or their combinations) are studied to show the generality of the proposed approach, with the ensuing results compared with existing data (if available) and with numerical FE results.

Keywords: Elastic stability; Simply-supported rectangular plates; Ritz's energy technique; Arbitrary loads; Buckling coefficients; Exact stress distributions; Double Fourier series

1. Introduction

Solutions to plate-buckling problems are usually obtained based on the assumption that the stress distribution throughout the plate is uniform [1,2]. Among the relatively small number of cases solved by allowing for nonuniform stress fields, the case of locally-distributed compressive patch loading on opposite edges of a plate [3–8] is, perhaps, the most common one. However, due to the inadequacy in choosing exact stress distributions [3–5] or true buckling profiles of plates [3,5–8], the results obtained by the previous studies cannot be said to be 'exact' or even accurate. Another case that has been studied involves plates under locally-distributed compression and equilibrating support shear. This problem was investigated by many authors [9–14], the energy method being often adopted, but, again, the in-plane stresses and buckled shapes used are often approximations to their true counterparts.

Clearly, an approach which is capable of predicting accurately the buckling of plates under arbitrary load conditions is desirable since, in practice, the external loads are usually nonuniform. Of course, one can solve such problems by recourse to numerical techniques such as the finite-element (FE) method. Indeed, previous studies using FE packages (ANSYS or ABAQUS) have addressed some such problems (though not fully) [15–17]. Analytical techniques, however, can be used as benchmarks and are also better suited for performing parametric studies which are often useful for the understanding of a problem.

This article shows how the stability of simply-supported rectangular plates under arbitrary stress distributions can be tackled in an accurate – indeed, practically 'exact' – manner. The elastic stability of simply-supported rectangular flat plates under arbitrary loads is presently investigated using the Ritz energy technique. By adopting the exact stresses [18–20] within a plate under any type of external loads and using the double Fourier series to represent any possible buckled profile, the buckling loads can be obtained very accurately. This paper can be seen as an extension and generalization of previous work by Pavlović and Baker [6–8] – in which the buckling of rectangular plates under locally-distributed direct stresses in the form of equal and opposite patch loads was studied – in order to cater for plate buckling under any form of external loads. The results produced by the general approach proposed in this paper are compared to those from the numerical FE method as well as to data available from earlier analyses.

* Corresponding author. Tel.: +44 20 75945986; fax: +44 20 75945989.
E-mail address: m.pavlovic@imperial.ac.uk (M.N. Pavlović).

0141-0296/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2007.07.025

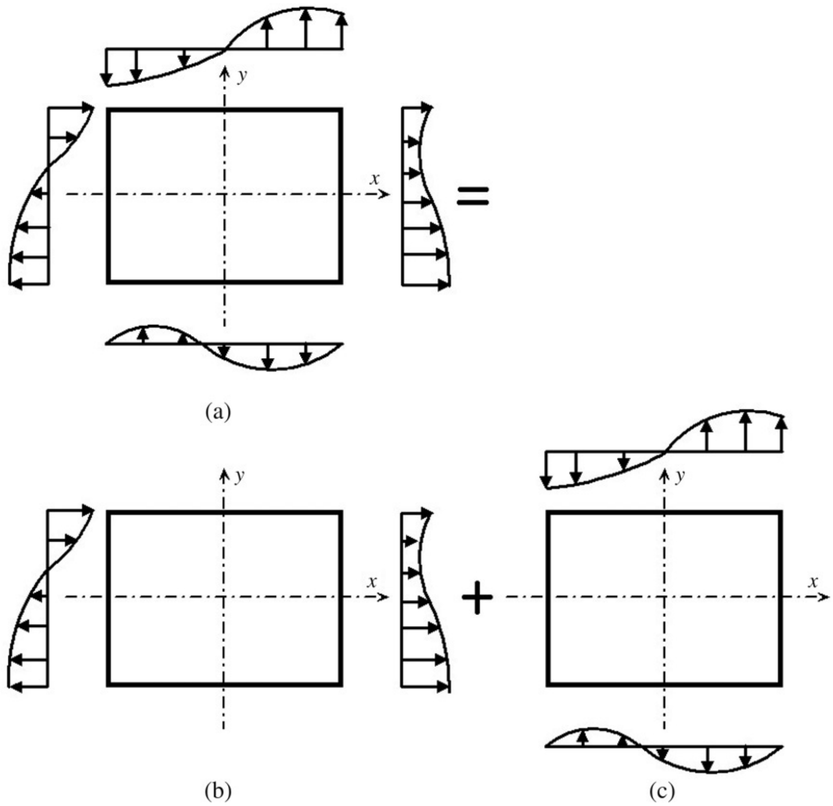

Fig. 1. Arbitrary direct stresses on four boundaries: case (a) is obtained by superimposing the direct stresses on the pair of boundaries in the vertical direction — case (b) — and the direct stresses on the pair of boundaries in the horizontal direction — case (c).

The paper begins with the determination of exact stresses within simply-supported rectangular plates using the mathematical solutions presented over the last hundred years [18–20]. Then, the formulation of the eigenvalue problem is outlined and an example is used to illustrate how the exact integration of the work done by the external loads is computed. Finally, the proposed analytical approach is applied to some practical load conditions which involve direct, bending and shear stresses so as to demonstrate its generality. In all cases, the ensuing analytical buckling coefficients are compared with existing data (when available), while their accuracy was reaffirmed by numerical (FE) results.

2. Exact stresses within rectangular plates

In this section, attention is paid to the exact stresses in rectangular plates under arbitrary edge loads. The breaking-down of arbitrary external loads into standard forms is first introduced to facilitate the use of the exact solutions to two-dimensional (2-D) elasticity problems outlined in previous publications [18–20]. This is followed by the use of these 2-D exact solutions to find the stresses for systems which are orientated at 90° to the fundamental cases. Finally, the exact stresses within the plate are found through the superposition method since linearly elastic assumptions hold throughout the present work.

2.1. Breaking-down of external loads and coefficients A_n

Consider rectangular plates under the action of in-plane arbitrary boundary loads which produce some arbitrary distribution of the external forces on all four edges. In the general case, on each of these four edges, there are two types of stresses, namely normal stresses (denoted as X or Y) and shear stresses (denoted as S or T). Furthermore, let $X^+(y)$, $X^-(y)$, $Y^+(x)$ and $Y^-(x)$ represent the normal stresses on the edges $x = \frac{1}{2}a$, $x = -\frac{1}{2}a$, $y = \frac{1}{2}b$ and $y = -\frac{1}{2}b$, respectively, as shown in Fig. 1(a): these functions are able to represent arbitrary load conditions that can be applied to the plate under consideration. (In what follows, only direct external stresses will be discussed in detail, but the procedure is equally applicable to shear stresses [20].)

The problem in Fig. 1(a) for direct stresses is represented by the superposition of the two elasticity problems in Fig. 1(b) and (c). It is easily seen that, if the problem in Fig. 1(b) is solved, it is quite straightforward to also solve the problem in Fig. 1(c) since the stresses for one problem can be obtained by simply rotating the original coordinate system by $\frac{1}{2}\pi$ clockwise or counter-clockwise.

It can be concluded, therefore, that what is important is to solve the general problem in Fig. 1(b) for normal stresses, in which the boundary conditions are defined by the external loads $X^+(y)$ and $X^-(y)$. To tackle the problems in a neat way, the external loads must be broken down further to take advantage