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Green's function solution for transient heat conduction in concrete-filled CHS subjected to fire

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Abstract

The heat transfer analysis of concrete-filled steel circular hollow sections (CHS) subjected to fire is essentially classified as the process of deriving a solution for transient heat conduction in a composite medium in a cylindrical coordinate system. In this paper, an efficient numerical approach based on an analytical Green's function (GF) solution is developed, where the spatial variable is analytically tractable without recourse to spatial discretization. The thin steel layer is conveniently treated as a lumped heat capacitance with uniform temperature distribution to avoid tedious analytical solutions for hollow cylinders. A computational model is developed based on Duhamel's principle, incorporating time-varying fire conditions. The choice of sampling time is not constrained by numerical stability and can be as large as 30 min in one time step. The numerical approach can be used to predict the temperature field inside the entire composite domain and the heat flux at the fire and the steel-concrete interfaces, as well as to verify more sophisticated numerical codes, e.g. finite element codes, of heat transfer.

Keywords: Transient; Heat conduction; Green's function; Concrete-filled CHS; Fire

1. Introduction

Due to the world-wide impetus in the performance-based design approach to buildings, research on structure fire analysis has become increasingly important in recent years. In current design codes [1–3], structural fire resistance is conventionally determined by assessing the load-bearing capacity of the structural components subjected to fire conditions, where the temperature field inside the structural members is largely approximated by empirical relations. Accurate temperature prediction of structural members subjected to fire conditions is of paramount importance for a safe and reliable structural fire design. Temperature prediction for structures in fire can essentially be classified as the solution to transient heat conduction problems in different structural components. There are a number of benchmark publications [4–7] with extensive discussions on different mathematical techniques for solutions of transient heat transfer in solid domains in rather general problem settings. An in-depth review of mathematical methods in heat transfer problems was given by Kumar [8].

When time-varying fire conditions are considered, a few sets of handy formulations for temperature prediction of uniformly insulated steel I-sections were obtained using different mathematical techniques, including the separation of variables (eigenfunction approach) [9], Laplace transform [10], and Green's function approach [11]. In these formulations [9–11], the main assumption is that the entire steel section is treated as a lumped heat capacitance and therefore the temperature distribution is uniform due to the high conductivity of steel. This concept has been shown to be very useful in solving heat conduction within protected steel members in fire and is valid in most of the structure-fire scenarios.

Eurocode 3 [1] adopted the time-discrete form of the SP approach [9] developed by Wickstrom, where numerical stability dictates the choice of sampling time, normally $\Delta t \leq 30$ s. A sensitivity study was conducted by Wang and Tan [12] to study the time delay coefficients whereby an appropriate range of applicability was imposed for the SP approach [9] and EC3 [1]. For the temperature prediction of concrete-encased I-sections, Wang and Tan [13] proposed the "residual area method", with the composite domain treated virtually as a

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Nomenclature

A	constant defined in Eq. (41)
$B(n), C(n)$	functions defined in Eqs. (42) and (43), respectively
Bi	Biot number (-)
c	specific heat ($J/\text{kg}\ K$)
C_n	coefficients in Fourier series solution of heat conduction
F	step Green's function at concrete–steel interface ($\text{m}^2\ \text{K/W}$)
$Fo, \bar{t}$	Fourier number or dimensionless time (-)
$g(r, t-\tau)$	impulse Green's function in cylindrical system ($\text{m}^2\ \text{K/J}$)
$G(r, t)$	step Green's function ($\text{m}^2\ \text{K/W}$)
h_{comb}, h_c	coefficient of combined and convective surface heat transfer, respectively ($\text{W/m}^2\ \text{K}$)
$H(\cdot)$	Heaviside function
$J_n(\cdot)$	Bessel function of the first kind of order n
k	thermal conductivity (W/m K)
L_s, t_s	characteristic and physical thickness of steel CHS, respectively (m)
q	heat flux (W/m^2)
Q	heat capacitance (per unit area) (J/K m^2)
r	coordinates in cylindrical system (-)
r_0, r_1	inner and outer radius of CHS, respectively (m)
$S_n(\upsilon, \Delta F)$	function defined in Eq. (38)
t	time (s)
$T(r, t)$	temperature field as a function of space and time (°C)
T_0	initial temperature field inside a structural member (°C)
T_g, T_s	temperature of fire gas and steel CHS, respectively (°C)
$V(r, t)$	characteristic step function (-)

Greek letters

α	thermal diffusivity (m^2/s)
$\delta(\cdot)$	Dirac delta function
Δt	sampling time (s)
ε_{res}	resultant emissivity for radiative heat transfer
θ	normalized temperature (°C)
ρ	density (kg/m^3)
σ	Stefan–Boltzmann constant ($\text{W/m}^2\ \text{K}^4$)
τ	time (s)
ξ	spatial points where heat impulse occurs in a homogeneous system (m)
ζ_n	coefficients in Fourier series solution

Subscripts

c	property of concrete
j	index of time stepping
n	summation index
g	property of fire gas
s	property of steel

Superscript

i	i-th iteration

contour-protected I-section and the remaining part as a residual area of concrete.

More sophisticated numerical codes for solving the transient heat conduction problems in structure fire design are available, including the finite difference method [14] and finite element programs, such as TASEF [15] and HEAT2D [16]. When computational cost and other practical issues such as availability, ease in implementation and stability of solutions are concerned, formulations based on analytical solutions are always preferred in temperature analysis. Among all the mathematical techniques available for transient heat conduction problems, the Green's function approach [6,11,17] is a systematic solution procedure that makes use of alternative forms of small and large time solutions to improve the convergence of non-homogeneous problems. The property of fast convergence of solutions in any time interval makes the Green's function approach a powerful and preferable technique for the solution of transient heat conduction problems.

The derivation of solutions for transient heat conduction in concrete-filled CHS in fire is particularly difficult due to the composite nature of the problem domain and the choice of a cylindrical coordinate system. Thus, a handy formulation for the temperature prediction of concrete-filled CHS based on analytical solutions has hitherto been absent from the literature. In this paper, analytical formulations and a numerical model are developed to tackle the temperature analysis for concrete-filled CHS based on Green's functions. The steel section is conveniently treated as a lumped capacitance with uniform temperature. The thermal properties for both steel and concrete are assumed to be isotropic and temperature-independent. No volumetric heat generation is involved in the heat transfer model. The steel-concrete interface is assumed to be in perfect contact. The length is assumed to be large enough compared to its radius and subjected to uniform heating conditions, such that there is no heat transfer in the longitudinal direction.

The outline of this paper is as follows. First of all, the heat transfer model for the concrete-filled CHS subjected to fire conditions is defined. Then the concept of the homogeneous systems and the step systems corresponding to the impulse and the step Green's functions is clarified. This is followed by evaluation of the impulse and the step Green's functions. Duhamel's principle is then discussed and employed to incorporate time-varying fire conditions. After that, the development of a the numerical model to implement the Green's function solution in terms of the discrete convolution integration with time discretization is detailed. The solution procedure is then outlined for convenience of implementation. Lastly, the temperature predictions by the Green's function solution are compared with the results obtained from a finite element code.

2. Definition of heat transfer models

For an isotropic concrete-filled CHS, as shown in Fig. 1, with the outer surface subjected to equal fire exposure, the temperature distribution will be dependent on only one spatial variable, namely the radial coordinate r. Let $T(r, t)$ denote the