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Benchmark hydroelastic responses of a circular VLFS under wave action

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Abstract

This paper is concerned with the hydroelastic analysis of a pontoon-type circular very large floating structure (VLFS). The coupled fluid–structure interaction problem may be solved by using the modal expansion method in the frequency domain. It involves, firstly, the decomposition of the deflection of a circular Mindlin plate with free edges into vibration modes which can be obtained in an exact manner. Then the hydrodynamic diffraction and radiation forces are evaluated by using eigenfunction expansion matching method that is also done in an exact manner. The hydroelastic equation of motion is solved by the Rayleigh–Ritz method for the modal amplitudes, and then the modal responses are summed up to obtain the total response. The very accurate deflections and stress-resultants of uniform circular VLFSs presented herein are valuable as they serve as benchmark solutions to check the numerical methods for hydroelastic analysis of VLFSs.

Keywords: Circular plate; Free vibration; Hydroelastic analysis; Modal expansion method; Very large floating structures; Mindlin plate theory

1. Introduction

The hydroelastic analysis of pontoon-type, very large floating structures (VLFSs) has attracted the attention of many Japanese engineers and researchers working in the offshore construction industry. This interest was heightened by the building of large floating oil storage facilities off the coasts of Kamigoto and Shirashima Islands and more so by the construction of the 1 km long test runway (commonly referred to as the Mega-Float) in Tokyo Bay in 1998. Such floating structures are very flexible and the elastic deformations due to wave action are more crucial than the rigid body motions. Many papers published on the hydroelastic analysis of VLFSs focus on a rectangular planform, mainly because it is the basic shape for construction and its vibration modes may be approximated by the products of natural modes of free–free beams that satisfy the natural boundary conditions (see for example Ref. [1]). A VLFS of a general planform shape requires numerical methods for analysis, and the boundary element method and finite element method are frequently used. As performing a rigorous hydroelastic analysis requires an enormous computational effort, researchers have proposed various techniques to improve the efficacy of the boundary element and finite element method [2–8]. Kashiwagi [9] and Watanabe et al. [10] presented a review of these methods.

In order to validate these numerical methods and to check the accuracy and convergence of the hydroelastic responses, analytical solutions are vitally needed. To this end, we consider circular VLFSs as accurate/exact analytical solutions may be obtained for benchmarking purposes. Hamamoto and Tanaka [11] and Hamamoto [12] were early researchers working on circular VLFSs. They developed an analytical approach to predict the dynamic response of a flexible circular floating island subjected to stochastic wind-waves and seaquakes. Zilman and Miloh [13] obtained closed form solutions for the hydroelastic response of a circular floating thin plate in shallow water. Tsubogo [14,15] solved the same floating circular plate problem independently. However, the assumption of shallow water limits the applicability range. The extension of Zilman and Miloh's method to a finite water depth was recently made by Peter et al. [16].

In this paper, we derive the governing equations and analytical expressions for the hydroelastic responses of a circular VLFS under wave action, considering a finite water depth and the effects of transverse shear deformation and

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rotary inertia on the plate deformation. Earlier aforementioned studies adopted the classical thin plate theory that neglects these effects. The presented accurate hydroelastic responses for the circular VLFS should be extremely useful as benchmark solutions in the development of techniques and software for hydroelastic analysis.

In our approach for analysis, the coupled fluid–structure interaction problem may be solved by firstly decomposing the unknown deflection of the plate (modeled by the Mindlin plate theory) into modal functions associated with a freely vibrating plate in air. The modal functions are obtained in an exact manner by solving analytically the governing equations of a vibrating Mindlin circular plate in dry air. The second step involves substituting the exact modal functions into the hydrodynamic equations and solving the boundary value problem using the eigenfunction expansion matching method which is also an analytical approach. The modal amplitudes for the set of equations of motion obtained are then back substituted into the modal functions and the stress-resultants for the deflections and stress-resultants of the VLFS. Two numerical examples are given to illustrate the method and the accurate results for deflections and stress-resultants obtained are presented graphically with their peak values tabulated for easy comparison when developing numerical methods and software for hydroelastic analysis of VLFSs.

2. Basic assumptions, equations and boundary conditions for a circular VLFS

In a basic hydroelastic analysis of pontoon-type VLFSs, the following assumptions are usually made:

- The VLFS is modeled as a flat plate with free edges.
- The fluid is incompressible, inviscid and its motion is irrotational so that the velocity potential exists.
- The amplitude of the incident wave and the motions of the VLFS are both small and only the vertical motion of structure is considered.
- There are no gaps between the VLFS and the water surface.

The fluid–structure system and the cylindrical coordinate system are shown in Fig. 1. The origin of the coordinate system is on the flat seabed and the z-axis is pointing upwards. The undisturbed free surface is on the plane $z = d$, and the seabed is assumed to be flat at $z = 0$. The floating, flat, circular plate has a radius of R and a uniform thickness h. The zero draft is assumed for simplifying the fluid-domain analysis. The sinusoidal plane wave is assumed to be incident at $\theta = 0$. The problem at hand is to determine the deflections and stress-resultants of the uniform circular plate under the action of the incident wave. Below, the governing equations and boundary conditions for the hydroelastic analysis are presented. The hydroelastic analysis is performed in the frequency domain.

Based on the second assumption given above and considering time-harmonic motions with the complex time dependence $e^{i\varpi t}$ being applied to all first-order oscillatory quantities, where i represents the imaginary unit, ϖ the angular

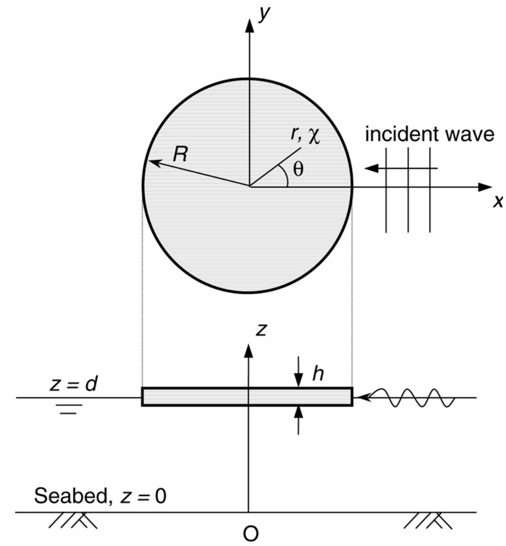

Fig. 1. Geometry of a uniform circular VLFS and coordinate system.

frequency of the wave and t the time, the complex velocity potential $\phi(r, \theta, z)$ is governed by the Laplace equation:

$$\nabla^2\phi(r, \theta, z) = 0 \tag{1}$$

in the fluid domain. The potential must satisfy the following boundary conditions on the free surface, on the seabed, and on the wetted bottom surface of the floating body:

$$\frac{\partial\phi(r, \theta, z)}{\partial z} = \frac{\varpi^2}{g}\phi(r, \theta, z) \qquad \text{on } z = d, r > R \tag{2}$$

$$\frac{\partial\phi(r, \theta, z)}{\partial z} = 0 \qquad \text{on } z = 0 \tag{3}$$

$$\frac{\partial\phi(r, \theta, z)}{\partial z} = \mathrm{i}\varpi w(r, \theta) \qquad \text{on } z = d, r \le R \tag{4}$$

where $w(r, \theta)$ is the vertical complex displacement of the plate, and g the gravitational acceleration.

The radiation condition for the scattering and radiation potential is also applied at infinity.

$$\lim_{r\to\infty}\sqrt{r}\left[\frac{\partial(\phi - \phi_I)}{\partial r} + \mathrm{i}k(\phi - \phi_I)\right] = 0 \qquad \text{as } r \to \infty \tag{5}$$

where r is the radial coordinate measured from the centre of the circular VLFS, k the wave number, and ϕ_I the potential representing the undisturbed incident wave:

$$\phi_I = \frac{\mathrm{i}gA\cosh kz}{\varpi\cosh kd}e^{\mathrm{i}kx} = \frac{\mathrm{i}gAM_0^{1/2}}{\varpi\cosh kd}f_0(z)\sum_{n=0}^{\infty}\varepsilon_n \mathrm{i}^n J_n(kr)\cos n\theta \tag{6}$$

where $\varepsilon_0 = 1, \varepsilon_n = 2$ $(n \ge 2)$; A is the amplitude of the incident wave; J_n is the Bessel function of the first kind of order n; and

$$k\tanh kd = \frac{\varpi^2}{g} \tag{7}$$

$$f_0(z) = M_0^{-1/2}\cosh kz \tag{8}$$