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Engineering Structures 28 (2006) 1197–1208
ENGINEERING STRUCTURES
www.elsevier.com/locate/engstruct

Correlation coefficients for structures with viscoelastic dampers

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Received 17 February 2005; received in revised form 12 December 2005; accepted 13 December 2005
Available online 20 March 2006

Abstract

The correlation coefficients of SDoF oscillators play a fundamental role in the seismic analysis of structures. These coefficients, in fact, are required in order to properly combine the modal maxima in the Lagrangian space, for example via the complete quadratic combination (CQC) rule. In the framework of the random vibration of linear systems, alternative frequency- and time-domain approaches to evaluate the correlation coefficients of SDoF oscillators with viscoelastic memory are presented, and validated by means of Monte Carlo simulation. In contrast to the usual modal strain energy (MSE) method, the proposed formulations allow the seismic analysis of structures with added viscoelastic dampers to be performed in a consistent modal space, where the fading memory arising from these devices is not neglected. Moreover, the frequency content of the ground motion can be accounted for. The numerical applications show that the inaccuracy associated with the conventional analyses (MSE + CQC) may be unacceptable for engineering purposes.

Keywords: Correlation coefficients; Complete quadratic combination (CQC) rule; Modal analysis; Modal strain energy (MSE) method; Random vibration; Viscoelastic dampers

1. Introduction

In the last decades viscoelastic dampers have been successfully used with the aim of mitigating the structural vibrations induced by natural actions, such as seismic motions, wind gusts, or ocean waves [1]. Continuous improvements in the techniques of identification and analysis, in fact, paralleled by noticeable refinements of device hardware, made the use of viscoelastic dampers completely suitable for consideration in both new or retrofitted constructions.

Since one of the main goals in the design procedure of engineering structures is the reliable prediction of the response to selected loads, vast effort has been devoted to work out effective tools for the dynamic analysis of viscoelastically damped structures. Indeed, this is not an easy task, even under the hypothesis of a linear behaviour. In the time domain, in fact, the motion is ruled by integro-differential equations, involving the relaxation functions of the viscoelastic dampers. Depending on the mathematical representation of these devices, however, alternative state-space models have been proposed in order to reduce the computational burden [e.g. [2–4]]. Analyses in the frequency domain can be also carried out, in which the dependence of stiffness and dissipation on the vibration frequency is accounted for via the complex-valued dynamic stiffness of the viscoelastic dampers [5,6].

In engineering practice, nevertheless, rigorous analyses are avoided, and simplifying assumptions are employed in the design procedure: in particular, the classical mode superposition method is applied along with the response spectrum technique. Contrary to conventional structures, however, the approximate modal quantities involved in the analysis, i.e. modal shapes, undamped natural frequencies and viscous damping ratios, are computed by means of the so-called modal strain energy (MSE) method. The latter was originally proposed by Johnson and Kienholz [7] as a technique for the finite element analysis of laminates containing a viscoelastic layer. To the best knowledge of the author, the first application of the MSE method in Earthquake Engineering was proposed by Chang et al. [8], with the aim of assessing the equivalent viscous damping of a viscoelastically damped steel frame. A number of numerical and experimental studies have been successively published. Among these, Zambrano et al. [9] elucidated that the inaccuracy in the prediction

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0141-0296/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2005.12.015

of the dynamic response is smaller for classically damped structures, i.e. structures in which the equations of motion are uncoupled by real-valued modes. Afterwards, Tsai and Chang [10] investigated the effects of some assumptions made in deriving the MSE method, and concluded that “the difference arising from the assumptions becomes significant when the damping ratio is larger than 20%”. In the latter study, however, only the ideal case of the so-called linear hysteretic damping [e.g. [11–13]], i.e. rate-independent stiffness and dissipation, is considered. Given that in real life stiffness and dissipation of the viscoelastic devices vary with the vibration frequency, the actual inaccuracy of the MSE method may be much higher, even if the damping is lower. As an example, Palmeri and Ricciardelli [14] recently showed that the MSE method may dramatically overestimate the fatigue life of wind-exposed structures with added viscoelastic dampers.

As a result of applying the MSE method, the dynamic analysis of a structure provided with viscoelastic dampers is carried out on an equivalent structure, which is classically and viscously damped. Specifically, the assumption to be classically damped is approximately met when the distribution of the viscoelastic dampers is almost homogeneous, as is usual in most real cases. On the contrary, the definition of an equivalent viscous damping for the modal oscillators may lead to some degree of inaccuracy. In the first instance, the fading memory introduced in the structure together with the viscoelastic dampers is totally neglected in the analyses. In doing so, depending on the dynamic characteristics of excitation and structure–damper system, the response of the modal oscillators may be largely under- or over-estimated. Furthermore, when the classical complete quadratic combination (CQC) rule is applied, the use of the correlation coefficients available in the literature has to be carefully considered, since these coefficients have been derived under the assumption of a mere viscous damping.

In this paper, after some preliminary concepts on the dynamics of structures with viscoelastic dampers, alternative procedures to consistently evaluate the correlation coefficients of modal oscillators featuring a linear viscoelastic memory are proposed. Both frequency- and time-domain approaches are developed. In a first stage, a direct approach in the frequency domain is considered, which can be used with any coloured input. The main disadvantage of this formulation is that some integrals have to be numerically computed. In a second stage, a time-domain approach is proposed in the simplest case of white noise input, in which the correlation coefficients are directly obtained by solving a set of linear equations, made compact by means of Kronecker algebra [15,16]. In a third stage, finally, the latter approach is extended to the more realistic case of filtered white noise input, in which the correlation coefficients are obtained by solving in cascade more sets of linear equations.

The numerical examples herein included demonstrate that the inaccuracy associated with the use of the MSE method in conjunction with the CQC rule may be unacceptable for engineering purposes. The results given by the proposed formulations, on the contrary, are always in good agreement with those given by direct analyses. The effects on the correlation coefficients of the frequency content of the excitation and of the mechanical parameters of the modal oscillators are also highlighted.

2. Preliminary concepts

2.1. Equations of motion

The equations governing the seismic-induced motion of a linear MDoF structure with added energy dissipation devices (Fig. 1(a)) can be written as:

$$\mathbf{M}\,\ddot{\mathbf{y}}(t) + \mathbf{C}\,\dot{\mathbf{y}}(t) + \mathbf{K}\,\mathbf{y}(t) + \sum_{i=1}^{r} \mathbf{L}_i^{\mathrm{T}}\, f_i(t) = \mathbf{L}_{\mathrm{g}}\,\ddot{y}_{\mathrm{g}}(t) \tag{1}$$

where $\mathbf{y}(t) = \begin{bmatrix} y_1(t) & \cdots & y_n(t) \end{bmatrix}^{\mathrm{T}}$ is the array collecting the n Lagrangian coordinates; the over-dot means time derivative; $\mathbf{M}$ is the matrix of inertia; $\mathbf{K}$ and $\mathbf{C}$ are the matrices of elastic stiffness and viscous damping pertinent to the structure without dampers, respectively; $\ddot{y}_{\mathrm{g}}(t)$ is the ground acceleration, and $\mathbf{L}_{\mathrm{g}}$ is its influence vector; $f_i(t)$ is the internal force in the i-th of the r dampers incorporated in the structure, and $\mathbf{L}_i^{\mathrm{T}}$ is its influence vector.

When viscoelastic dampers are used, the force $f_i(t)$ at a generic instant $t > 0$ depends in principle on the whole time history of the associated deformation $d_i(s) = \mathbf{L}_i\,\mathbf{y}(s)$, with $0 \le s \le t$; that is, viscoelastic devices are systems with memory, as the knowledge of the usual state variables (displacements and velocities) is insufficient to predict their response. Under the assumption of a linear behaviour, the constitutive law of the i-th damper can be expressed in the time domain through the convolution integral [17]:

$$f_i(t) = \int_0^t \varphi_i(t-s)\,\dot{d}_i(s)\,\mathrm{d}s \tag{2}$$

in which the kernel $\varphi_i(t)$ is the relaxation function of the i-th viscoelastic damper. Upon substitution of Eq. (2) into Eq. (1), the dynamic equilibrium is governed by a set of n coupled integro-differential equations of second order:

$$\mathbf{M}\,\ddot{\mathbf{y}}(t) + \mathbf{C}\,\dot{\mathbf{y}}(t) + \mathbf{K}\,\mathbf{y}(t) + \sum_{i=1}^{r} \mathbf{L}_i^{\mathrm{T}} \int_0^t \varphi_i(t-s)\,\mathbf{L}_i\,\dot{\mathbf{y}}(s)\,\mathrm{d}s = \mathbf{L}_{\mathrm{g}}\,\ddot{y}_{\mathrm{g}}(t). \tag{3}$$

As an alternative, in the frequency domain Eq. (2) turns into:

$$\mathrm{F}\langle f_i(t)\rangle = K_i(\omega)\,\mathrm{F}\langle \delta_i(t)\rangle = K_i(\omega)\,\mathbf{L}_i\,\mathrm{F}\langle \mathbf{y}(t)\rangle$$

where $\mathrm{F}\langle\cdot\rangle$ stands for the Fourier transform operator, and $K_i(\omega) = (\mathrm{j}\,\omega)\,\mathrm{F}\langle\varphi_i(t)\rangle$ is the complex-valued dynamic stiffness of the i-th viscoelastic damper, $\mathrm{j} = \sqrt{-1}$ being the imaginary unit. Eq. (3), then, becomes in the frequency domain:

$$\left[-\omega^2\,\mathbf{M} + \mathrm{j}\left(\omega\,\mathbf{C} + \sum_{i=1}^{r} \mathbf{L}_i^{\mathrm{T}}\,K_i''(\omega)\,\mathbf{L}_i\right) + \left(\mathbf{K} + \sum_{i=1}^{r} \mathbf{L}_i^{\mathrm{T}}\,K_i'(\omega)\,\mathbf{L}_i\right)\right] \mathrm{F}\langle \mathbf{y}(t)\rangle = \mathbf{L}_{\mathrm{g}}\,\mathrm{F}\langle \ddot{y}_{\mathrm{g}}(t)\rangle \tag{4}$$