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Engineering Structures 28 (2006) 54–62

ENGINEERING STRUCTURES

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On fivefold coupled vibrations of Timoshenko thin-walled beams

A. Prokić*

Faculty of Civil Engineering Subotica, University of Novi Sad, Kozaračka 2a, Subotica, Serbia and Montenegro

Received 21 September 2004; received in revised form 12 July 2005; accepted 12 July 2005
Available online 26 August 2005

Abstract

The objective of the present paper is to analyze the motion of the Timoshenko thin-walled beam, with arbitrary open cross section, by means of an exact solution, and to study the influence of shear deformation over the natural frequencies. The effects of rotary inertia and warping stiffness are included in the present formulations. The five governing differential equations for coupled bending–torsional–shearing vibrations were performed using the principle of virtual displacements. The resulting coupling is referred to as fivefold coupled vibrations. In the case of a simply supported thin-walled beam, the closed-form solution for the natural frequencies of free harmonic vibrations was derived. The frequency equation, given in determinantal form, is expanded in an explicit analytical form, and then solved using the symbolic computing package *Mathcad 2001 professional*. In order to demonstrate the validity of this method the natural frequencies of asymmetric thin-walled beams having coupled deformation modes are evaluated and compared with analytical results analyzed by Vlasov theory.

Keywords: Vibration; Thin-walled beam; Timoshenko beam; Shear deformations

1. Introduction

Thin-walled beams of open cross section are widely used as structural components within the fields of civil engineering (bridge decks, core walls for tall buildings, ...), aerospace engineering (aircraft wings, propeller blades, ...) and mechanical engineering (turbine blades, vehicle axles, ...), offering a high performance in terms of minimum weight for a given strength. Vibration characteristics of those elements are of fundamental importance in the design of thin-walled structures.

Although Vlasov's theory of a thin-walled beam with open cross section is already well established, it presents limitations in the dynamic analysis of non-slender beams where the length-to-depth ratios are generally small, when higher natural frequencies are required, and also for composite beams when fibrous composites have low shear moduli which result in low shear stiffness of the beam. The range of applicability of the Vlasov's theory [1] can be extended by taking account of transverse shear deformations. The equations which include this effect are generally referred to as Timoshenko's beam equations.

Coupled bending–torsional vibrations of Timoshenko beams were studied by Bishop and Price [2], and Banerjee and Williams [3], who later took into account the effect of axial loading [4]. The warping effect was neglected in all of these works. More recently Bercin and Tanaka [5] took into account the warping stiffness to determine the free vibration modes of Timoshenko beams with monosymmetric open cross-section. A dynamic transfer matrix method of determining the natural frequencies and mode shapes of axially loaded thin-walled Timoshenko beams has been presented by Li et al. [6,7]. Vlasov's theory for the dynamic behaviour of thin-walled open section beams with warping were modified by Ambrosini et al. [8] in order to include the influence of shear flexibility and rotatory inertias. Relatively fewer studies are available that have been done toward the free vibration analysis of composite Timoshenko beams [9–11] and axially loaded composite Timoshenko beams [12,13].

In the general case of arbitrary cross sections of thin-walled beams, lateral and shearing vibrations, in two perpendicular directions, are coupled with torsional

* Tel.: +381 24 554227; fax: +381 24 554580.
E-mail address: aprokic@EUnet.yu.

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doi:10.1016/j.engstruct.2005.07.002

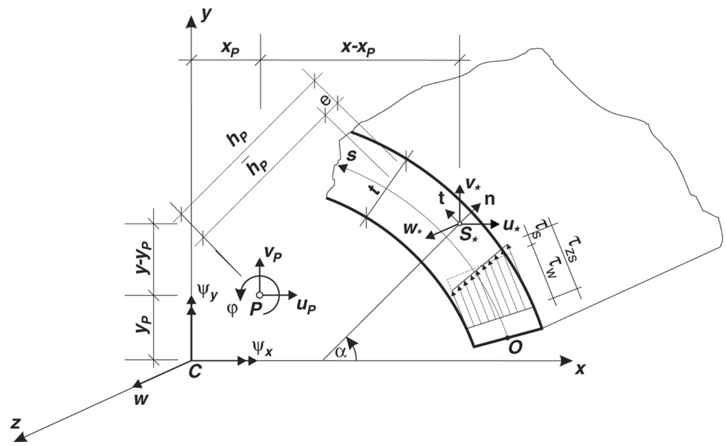

Fig. 1. Coordinate system and geometric parameters of a beam.

vibrations, and the frequency equations of such elements should be considered simultaneously. This situation may be termed fivefold coupling. In relation to Vlasov's theory there are two additional parameters, which are necessary to present shearing deformation. To the author's knowledge no studies are available that have been done toward investigating that case of coupled vibrations of Timoshenko thin-walled beams. This problem is presented in this paper. It is expected that the undertaken investigation will be useful for better understanding of dynamic characteristics of Timoshenko thin-walled beams.

2. Equations of motion

A straight thin-walled beam with arbitrary open cross-section is considered. Two coordinate systems are used. The first of these is a rectangular coordinate system x, y, z for which the z-axis coincides with the longitudinal centroidal axis, while x and y coincide with principal axes of the cross-section. The second coordinate system is a curvilinear coordinate system (e, s, z) where e and s are profile coordinates measured along the normal to the contour (middle line of a cross-section), and along the contour line, respectively. In addition to the usual assumptions of the linear theory of elasticity, the following assumptions are also made:

1. The cross-section is perfectly rigid in its own plane.
2. The part of the shear strains in the middle surface of the wall, due to the warping, is negligible.

According to the first assumption, the displacements u_* and v_* of an arbitrary point of cross section can be expressed in terms of componential displacements u and v of the shear center P as the pole and the rotation angle φ about the same pole (Fig. 1):

$$
\begin{aligned}
u_* &= u - \varphi\,(y - y_P) \\
v_* &= v + \varphi\,(x - x_P)\,.
\end{aligned} \tag{1}
$$

Using the second hypothesis the longitudinal displacement w_* of an arbitrary point of cross-section may be defined in the form that allows bending shear deformations

$$
w_* = w + \psi_x y - \psi_y x - \varphi' \omega \tag{2}
$$

where w represents the axial displacement of the centroid, ψ_x and ψ_y are cross-sectional rotations about x and y axes due to bending and ω is the so-called normalized sectorial coordinate or warping function. In Eq. (2), $(\,)'$ implies the differentiation with respect to z.

Component deformations for the defined displacement field are

$$
\begin{aligned}
\varepsilon_z &= \frac{\partial w_*}{\partial z} = \left(w' + \psi_x' y - \psi_y' x - \varphi'' \omega\right) \\
\gamma_{zx} &= \frac{\partial u_*}{\partial z} + \frac{\partial w_*}{\partial x} \\
&= \left[u' - \varphi'\,(y - y_P) - \psi_y - \varphi' \omega_{,x}\right] \\
\gamma_{zy} &= \frac{\partial v_*}{\partial z} + \frac{\partial w_*}{\partial y} = \left[v' + \varphi'\,(x - x_P) + \psi_x - \varphi' \omega_{,y}\right].
\end{aligned} \tag{3}
$$

For a linear elastic material the stress–strain relationship is defined by Hooke's law

$$
\begin{aligned}
\sigma_z &= E\left(w' + \psi_x' y - \psi_y' x - \varphi'' \omega\right) \\
\tau_{zx} &= G\left[u' - \varphi'\,(y - y_P) - \psi_y - \varphi' \omega_{,x}\right] \\
\tau_{zy} &= G\left[v' + \varphi'\,(x - x_P) + \psi_x - \varphi' \omega_{,y}\right].
\end{aligned} \tag{4}
$$

in which E is the modulus of elasticity and G the shear modulus.

The equations of motion of a thin-walled beam can be obtained using the principle of virtual displacements. A small element between cross sections $z_1 = z$ and $z_2 = z + \mathrm{d}z$ (Fig. 2) subjected to external loads $\bar{\boldsymbol{p}}(\bar{p}_x, \bar{p}_y, \bar{p}_z)$ per unit area of midplane is considered.

At any point on the cross section z_1 acts as a stress vector

$$
\boldsymbol{\sigma} = \tau_{zx} \boldsymbol{i}_x + \tau_{zy} \boldsymbol{i}_y + \sigma_z \boldsymbol{i}_z \tag{5}
$$

where $\boldsymbol{i}_x$, $\boldsymbol{i}_y$ and $\boldsymbol{i}_z$ are unit vectors of coordinate axes.

The vector of virtual displacements $\delta \boldsymbol{u}$, which satisfies the necessary continuity conditions and displacement boundary conditions, may be adopted in the same form as a vector of