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Engineering Structures 29 (2007) 483–497

ENGINEERING STRUCTURES

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# Proposal to evaluate the ultimate limit state of slender structures. Part 1: Technical aspects

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Received 9 December 2005; received in revised form 17 May 2006; accepted 19 May 2006
Available online 7 July 2006

**Abstract**

This paper presents a design method to evaluate the ultimate limit state in slender structures. The method addresses the main two features in this type of analysis: how to deal with the nonlinear geometric behaviour and the possible imperfections, incorporating all frame and member imperfections into a process that only requires cross-sectional checking. The method is based on a simplified second-order elastic analysis of the structure including lateral buckling with an equivalent geometric imperfection. The second order effects are calculated using the orthogonal properties of the buckling modes. The geometric imperfection is obtained from the buckling mode by a suitable scaling procedure using a generalization of Dutheil's method.


*Keywords:* Advanced analysis; Geometric nonlinearity; Design; Frames; Geometric imperfection; Ultimate limit state; Slender; Steel; Lateral buckling

## 1. Introduction

The optimum design using materials with a high strength–weight ratio results in slender structures (SS). SS behaviour is mainly affected by geometric and material nonlinearities.

### 1.1. Geometric nonlinearities

These structural effects, as already stated by Euler [16], cannot be ignored. Geometric nonlinearities include second-order effects associated with P–$\delta$ (non-sway) and P–$\Delta$ (sway) effects, and geometric imperfections.

Some design codes such as Eurocode 3 "EC-3" [6], consider these structural effects and propose the use of linear analysis with some interaction design formula for the calculation of geometric nonlinearities. Real structures may present several types of imperfections, for example:

1. Geometric imperfections (Fig. 1(a)): these are defined by the constructional tolerances specified in the design standards, and related to quality control during the construction process. The following list illustrates different types of geometric imperfections that can be found in a building structure:
   (a) at frame level: possible inter-storey drift due to tilting in columns
   (b) at member level: bowing out of straightness of the elements
   (c) at cross section level: variations in the dimensions (width, depth or thickness) along the element length or across the section, variations in the cover of the re-bars in concrete structures, etc.
2. Accidental eccentricities, caused by a slightly different loading position than originally considered.

Geometric and material nonlinearities can be analysed separately using the co-rotational formulation reported in Crisfield [10,11] based on the classic principle which consists of splitting the movements of the structure from the initial to the deformed configuration into two movements: a rigid body motion to the so-called co-rotated framework, and a deformational motion to the final deformed configuration.

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doi:10.1016/j.engstruct.2006.05.014

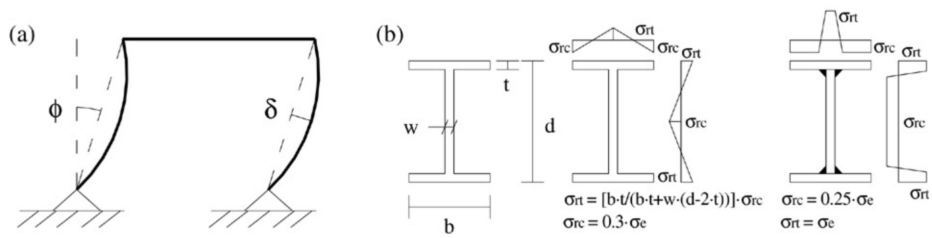


Fig. 1. Member imperfection ($\delta$), frame imperfection ($\Phi$) and residual stress diagrams.

### 1.2. Material nonlinearities

As mentioned above material nonlinearities are the second main factor affecting the behaviour of slender structures. These material nonlinearities include gradual yielding associated with flexural, torsional and axial stresses, as well as changes in material properties across the section and/or along the element length, and residual stresses, as reported by Nishino and Tall in [24] for steel structures (see Fig. 1(b)).

### 1.3. Design standards and simplified methods

All these imperfections and nonlinearities should be measured to perform an accurate analysis of the structure, but this can only be done after the construction has finished. Additionally, the stochastic nature of some of the factors affecting structural behaviour, such as loading or material properties, should also be taken into consideration (Marek et al. [22]).

However a design that accurately considers all these factors (material and geometric nonlinearities) requires high computational and economical costs, so simplified methods are needed; but this is a difficult task specially for the analysis of complex structural behaviour.

In this sense, current design codes simplify (or, at least, try to) analysis by considering only the key parameters in a simplified model but keeping model accuracy. The design procedure has to be simple, "clear", reliable and accurate as commented by Nethercot [23], avoiding the waste of building materials.

As an example, the current Spanish standard of concrete structures ("EHE" [15]) accounts for these structural factors at the member level by considering an *additional eccentricity* of the load, while the current Spanish steel standard "EA-95" [14] recommend the use of the coefficient "$\omega$" based on Dutheil's method [13] so that, implicitly, an *equivalent imperfection* is included.

Eurocode 3 [6] also recommends the *equivalent imperfection* approach giving the possibility of using the *notional load* method in sway structures. This Eurocode 3 proposes a second-order analysis of the imperfect structures using two types of imperfections: (a) member imperfections $\{\delta\}$, as discussed in Chan and Zhou [7], and (b) frame imperfections $\{\Phi\}$, as discussed in Clarke [9] for example. Sometimes the member imperfection can be neglected in the global analysis and considered by means of the suitable design formula with an auxiliary coefficient "$\chi$" as shown in Fig. 2, which displays the frame analysis process according to Eurocode 3.

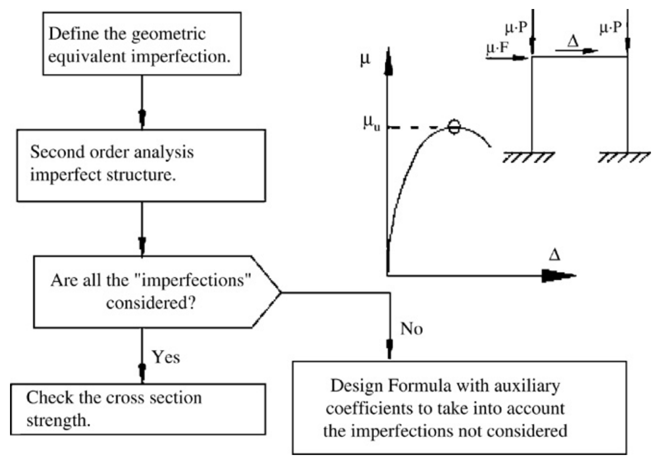


Fig. 2. Frame analysis according to Eurocode 3.

Other simplified methods use a *reduced modulus* approach (see Kim and Chen [20] and Wongkaew and Chen [37]) to account for the effects of material nonlinearities and geometric imperfections.

Taking all these aspects into consideration, a choice must be made when deciding on the complexity of the analysis and related auxiliary coefficients, because simple analyses always need very complex auxiliary coefficients (usually difficult to obtain for a general case making the formulation unclear and complicated) while complex analyses may not need auxiliary coefficients but have high computational costs.

Thus the development of simplified methods is a really difficult task. Some of the approaches and simplifications just mentioned suffer from the lack of some of the main basic properties. For example, buckling is a global problem and should be analysed taking into account the interaction of all the members of the structure rather than only the members with primary axial and strong bending moments. In contrast, many design codes like the concrete standard "EHE" [15], define an additional eccentricity per each beam–column at the member level instead of a general set of eccentricities at the structural level since this last choice should lead to the worst pattern of eccentricities. This can be clearly observed if the buckling problem is studied using a similar pattern of geometric imperfections instead of a pattern of eccentricities; several authors, such as Clarke [9], have studied the effect of imperfections on the behaviour of steel frames evidencing that the worst shape is given by the first buckling mode, which in turn affects the whole structure.