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ScienceDirect
Engineering Structures 29 (2007) 635–639
ENGINEERING STRUCTURES
www.elsevier.com/locate/engstruct

An exact truncation boundary condition for incompressible fluid domains in dam–reservoir interaction analysis

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Received 23 May 2005; received in revised form 25 April 2006; accepted 1 June 2006
Available online 24 July 2006

Abstract

In this paper, an exact truncation boundary condition is derived and implemented in a finite element code for the analysis of dam–reservoir interaction for incompressible, inviscid and unbounded fluid domains. The reservoir domain is divided into two regions in the derivation of the truncation boundary condition. These are the near field having a complex geometry and the far field with a uniform cross-section. The proposed boundary condition is obtained using the analytical solution for the far field and used as a truncation boundary condition at the truncation surface for the near field including the dam–reservoir system. This method has the advantage of geometrical flexibility in the near field and of being a semi-analytical approach giving close results to the exact solutions for the cases analyzed. In addition, accurate results are also obtained when the reservoir domain is truncated very close to the dam–reservoir interface.

Keywords: Truncation boundary; Dam-reservoir interaction; Finite element; Incompressible fluid; Hydrodynamic pressure

1. Introduction

In the finite element analysis of dam–reservoir interaction problems difficulties arise due to unbounded reservoir domain. This difficulty is handled by truncating the unbounded fluid domain at a certain distance away from the dam–reservoir interface. However, for an accurate analysis, the behavior of reservoir fluid at the truncation surface must be truly represented. Therefore, a suitable boundary condition is required along the truncation boundary.

Zienkiewicz et al. [1] studied the coupled response of structures submerged in incompressible fluids using the finite element method. Nath [2] presented a solution to the problem neglecting radiation damping. Chakrabarti and Chopra [3] assumed the reservoir was a continuum with infinite length. Chwang and Housner [4,5] analyzed the added-mass effect of horizontal acceleration in the dam–reservoir interaction both analytically and using a momentum balance approach.

Several boundary conditions have been proposed in the past. The most commonly used truncating boundary condition is the Sommerfeld radiation condition [6]. This boundary condition becomes a rigid stationary boundary for incompressible fluid domains and as a result does not represent the actual behavior of the reservoir domain. Another boundary condition is proposed by Sharan [7]. The Sharan boundary condition is obtained by using the exact solution of the reservoir fluid responses for a vertical faced rigid dam to represent the fluid behavior at a sufficiently large distance away from the dam–reservoir interface. Aviles and Sanchez-Sesma [8] proposed an analytical solution for dam–reservoir systems with non-vertical interface by using the Trefftz-Mikhlin method. This method produces an infinite set of algebraic equations in which the unknowns are the coefficients of an exact solution. Lately Küçükarslan [9] have proposed a truncating boundary condition by using the exact solution for hydrodynamic pressures for dam–reservoir systems with vertical dam–reservoir interface. This boundary condition is suitable for the analysis of dam–reservoir systems with dams having vertical upstream face.

The objective of the present study is to propose an exact truncation boundary condition for dam–reservoir interaction problems in unbounded fluid domains. The reservoir fluid domain is assumed to be incompressible and inviscid. In the derivation of an exact boundary condition, the reservoir domain

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0141-0296/$ - see front matter
doi:10.1016/j.engstruct.2006.06.005

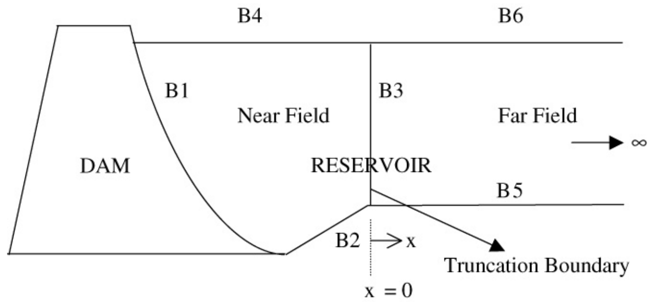

Fig. 1. Dam-reservoir system.

is divided into two regions. One is far field, which is uniform for the derivation of the truncation boundary condition and the other is near field including the dam–reservoir system in any geometry to be analyzed using the derived boundary condition from the far field. The main advantage of this approach is to gain a geometrical flexibility for the dam–reservoir system to be analyzed in the near field. Another advantage is to have an exact boundary condition on the truncation boundary, which leads to a semi-analytical solution. If the reservoir geometry is uniform in the near field, the presented formulation allows the truncation boundary to be in close vicinity to the dam.

2. Formulation of the problem

In the analysis of unbounded reservoir fluid domains, the reservoir is divided into two regions: the near field having a complex geometry and the far field with a uniform cross-section as shown in Fig. 1.

Truncation boundary condition is derived from the governing equation of hydrodynamic pressure in the far field. Hydrodynamic pressure satisfies the Laplace equation for an incompressible and inviscid fluid.

$$\nabla^2 p = 0. \tag{1}$$

Boundary conditions at the boundaries of the far field are:

On the truncation surface (B3),

$$\frac{\partial p}{\partial n}(x_{B3}, y) = -\frac{\partial p^f}{\partial n}(x_{B3}, y). \tag{2}$$

At the bottom of the far field (B5), if only horizontal ground motion is considered,

$$\frac{\partial p^f}{\partial n}(x, y) = 0. \tag{3}$$

At the far end where the x coordinate is infinite

$$p^f(\infty, y) = 0. \tag{4}$$

At the free surface (B6), if the effect of surface waves is neglected,

$$p^f(x, H) = 0 \tag{5}$$

where H is reservoir height in the far field and superscript 'f' represents the variable in the far field domain.

The analytical solution for hydrodynamic pressure, p^f, in the far field is:

$$p^f(x, y) = \sum_{k=1}^{\infty} A_k e^{-\lambda_k \frac{x}{H}} \cos\left(\lambda_k \frac{y}{H}\right) \tag{6}$$

where $\lambda_k = \frac{2k-1}{2}\pi$.

Pressure along the truncation surface is a function of y only and may be obtained by taking $x = 0$.

$$p^f\Big|_{x=0} = p^f(y) = \sum_{k=1}^{\infty} A_k \cos\left(\lambda_k \frac{y}{H}\right) \tag{7}$$

where

$$A_k = \frac{2}{H}\int_0^H p^f(y)\cos\left(\lambda_k \frac{y}{H}\right) \mathrm{d}y. \tag{8}$$

Once A_k is determined, the normal derivative of hydrodynamic pressure in the far field along the truncation boundary can be evaluated from the following equation.

$$\frac{\partial p^f}{\partial n} = -\frac{\partial p^f}{\partial x}. \tag{9}$$

At $x = 0$, the normal derivative of pressure in the far field in Eq. (9) becomes:

$$\frac{\partial p^f}{\partial n}\bigg|_{x=0} = \sum_{k=1}^{\infty} \frac{\lambda_k}{H} A_k \cos\left(\lambda_k \frac{y}{H}\right). \tag{10}$$

The required truncation boundary condition may be obtained by inserting Eq. (10) into Eq. (2) for the analysis of the dam–reservoir interaction considering the dam and near field. This boundary condition is:

$$\frac{\partial p}{\partial n}\bigg|_{B3} = \sum_{k=1}^{\infty} -\frac{\lambda_k}{H} A_k \cos\left(\lambda_k \frac{y}{H}\right). \tag{11}$$

The normal derivative of pressure in Eq. (11) is also equal to the derivative of pressure with respect to (x). The A_k's must be determined to complete the derivation of the boundary condition. Recall Eq. (8):

$$A_k = \frac{2}{H}\int_0^H p^f(y)\cos\left(\lambda_k \frac{y}{H}\right) \mathrm{d}y.$$

The truncation boundary is divided into 'n' equal parts as shown in Fig. 2. If nodal pressures along this boundary are assumed to change linearly, variation of the pressure along the ith part at the boundary (B3) may be expressed as follows:

$$p_i^f(y) = \left(i - \frac{y}{\Delta y}\right) p_{i-1}^f + \left(\frac{y}{\Delta y} - (i-1)\right) p_i^f \quad i = 1, 2, \ldots, n. \tag{12}$$

Eq. (8) may be rearranged by calculating the integral on the boundary (B3) as the summation of integrals in each divided part of this boundary.

$$A_k = \frac{2}{H}\sum_{i=1}^{n}\int_{y_{i-1}}^{y_i} p_i^f(y)\cos\left(\lambda_k \frac{y}{H}\right) \mathrm{d}y. \tag{13}$$

Substituting Eq. (12) into Eq. (13) one has A_k as: