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Limit analysis of a single span masonry bridge with unilateral frictional contact interfaces

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Abstract

In this paper is studied the ultimate failure (limit) load of stone arch bridges. The proposed model is based on finite element analysis with interfaces, simulating potential cracks, which allow for unilateral contact with friction. Opening or sliding of some interface indicates crack initiation. The ultimate load has been calculated by using a path-following (load incrementation) technique. Lack of a solution at a certain level of loading indicates onset of failure. For the validation of the proposed method, which is based on the contact model, the ultimate failure load is recalculated by using a modern implementation of the classical collapse mechanism method based on linear programming. Finally, the beneficial effect of the fill on the limit load of a real bridge is estimated and compared with experimental results.

Keywords: Masonry arch; Limit analysis; Unilateral contact; Friction

1. Introduction

Stone arch bridges are part of the cultural heritage of both Greece and many other countries. Many of them still survive, therefore a more detailed analysis of these monuments, including restoration, is of interest. The properties of the material and of the structure make this effort quite demanding. A stone bridge consists of stone blocks and mortar joints. Blocks have high strength in compression and low strength in tension while mortar has generally low strength. Thus a safe assumption of a no-tension material can be adopted at least for the purpose of limit analysis. Stone blocks and mortar have also different Young's moduli. In some cases the mortar does not exist (dry masonry). The mentioned variation in the mechanical properties of the bridge's materials leads to the development of a number of theories in order to represent as accurately as possible, the real mechanical behavior of the stone bridge.

Two widespread methods for the assessment of masonry arch bridges have been used in the past. The first, known as the Military Engineering Experimental Establishment (MEXE) method, is a semi-empirical one [1] and will not be considered further here, while the second is the collapse mechanism method proposed by Heyman [2,3], and its extensions. The limit analysis of block structures with a frictional contact interface law offers an interesting aspect in the study of masonry bridges. Several computational methods have been developed for the evaluation of the limit load of masonry structures. Drucker [4] first underlined the problem of applying the bound theorems of plasticity to frictional problems. Livesley [5] attempted to solve the problem of the collapse load evaluation of structures formed with frictional materials, as a linear programming problem. In Ref. [5] it has been demonstrated that the adoption of a simplified associated constitutive law may yield an overestimate of the true collapse load. Melbourne and Gilbert [6,7] confirmed that frictional assumptions are very important in multiring arches. Fishwick [8] developed automatic numerical schemes for the limit analysis of rigid block structures involving friction, while Baggio and Trovalusci [9] proposed mathematical

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programming approaches for carrying out the same task. Ferris and Tin-Loi [10] computed the collapse load of discrete rigid block systems with frictional contact interfaces as a special constrained optimization problem (the so-called mathematical problem under equilibrium constraints, MPEC). Fishwick et al. [11] formulated the limit analysis problem as an optimization problem and suggested a solution which involves the use of a genetic algorithm. Modern methods based on finite element analysis have also been developed for the study of masonry structures [12]. The models which have been developed in the past can be roughly divided into two large categories:

(1) Discrete models
The structure is divided into large discrete parts such as stone arch parts. The behavior of the contact surface between them is described by a unilateral law, possibly with friction, while the discrete elements are assumed to behave elastically. In the limit the parts are simply considered to be rigid bodies.
(2) Continuum models
The mechanical behavior of these models is described by a nonlinear constitutive law, where either:
   (a) the masonry is assumed to consist of a single material and its behavior is described by an inelastic theory (for instance an appropriately modified elastic-plastic model with fracture) [13], or:
   (b) the different mechanical behavior between stone and mortar and the anisotropy induced by them are taken into account on the basis of a homogenization theory [14].

In the present work the ultimate failure load of a stone arch bridge is found by the use of a discrete model formulation. In particular, the geometry of the structure is divided into a number of interfaces, perpendicular to the center line of the ring. Those interfaces are uniformly distributed in the arch. A parametric investigation concerning the interaction between their number and the ultimate load takes place; finally a large number of them is considered (see Section 5). Unilateral contact law governs the behavior in the normal direction of an interface, indicating that no tension forces can be transmitted in this direction. The behavior in the tangential direction takes into account that sliding may or may not occur. For the validation of the obtained results, the ultimate failure load is recalculated using a modern implementation of the traditional method of collapse mechanism, which is based on linear programming [6, 7]. A second, more complicated analysis in which the backfill is included is also presented. This solution is then compared with experimental data taken from the published literature. It should be mentioned that the method proposed here can be used for the limit load analysis of every multi-body structure in civil engineering (e.g. dry masonry walls, cracked rocks etc.) and beyond (e.g. gripper analysis in robotics).

2. The unilateral contact-friction model

2.1. The unilateral contact

Let us consider a point lying on the boundary of an elastic body which comes in contact with a rigid wall. Let u be the single degree of freedom of the system, g be the initial opening and t^n be the corresponding contact pressure in case contact occurs. The basic unilateral contact law is described by the set of inequalities (1), (2) and by the complementarity relation (3), [15–17]

$$h = u - g \leq 0 \Longrightarrow h \leq 0 \quad (1)$$

$$-t^n \geq 0 \quad (2)$$

$$t^n(u - g) = 0. \quad (3)$$

Inequality (1) represents the non-penetration relation, while relation (2) implements the requirement that only compressive stresses (contact pressures) are allowed. Eq. (3) is the complementarity relation which states that either separation with zero contact stress occurs or contact is realized with possibly non-zero contact stress. For a discretized structure the previous relations are written for every point of a unilateral boundary or interface by using appropriate vectors, as will be mentioned later in this paper.

2.2. Frictional modelling

The behavior in the tangential direction is defined by a static version of the Coulomb friction model. Two contacting surfaces start sliding when the shear stress in the interface reaches a critical value equal to:

$$t^t = \tau_{cr} = \pm\mu|t^n| \quad (4)$$

where t^t, t^n are the shear stress and the contact pressure at a given point of the contacting surfaces respectively and μ is the friction coefficient. There are two possible directions of sliding along an interface, so t^t can be positive or negative depending on that direction. Furthermore, there is no sliding if $|t^t| < \mu|t^n|$ (stick conditions). The sliding rule can be summarized by the following relations

$$|t^t| < \mu|t^n| \longrightarrow u_t = 0 \text{ (no sliding)} \quad (5a)$$

$$t^t = \mu|t^n| \longrightarrow u_t \geq 0 \text{ (sliding in one direction)} \quad (5b)$$

$$t^t = -\mu|t^n| \longrightarrow u_t \leq 0 \text{ (sliding in the opposite direction)} \quad (5c)$$

where u_t is the displacement (sliding) in the tangential direction of an interface. In order to express the frictional rules with complementarity relations, slack variables are introduced (Fig. 1) and the problem is written as follows

$$t^t + r_1 = \mu|t^n| \quad (6a)$$

$$t^t - r_2 = -\mu|t^n| \quad (6b)$$

$$u_t = \lambda_1 - \lambda_2 \quad (6c)$$

$$r_1\lambda_1 = 0, \quad r_2\lambda_2 = 0 \quad (6d)$$

$$r_1, r_2, \lambda_1, \lambda_2 \geq 0. \quad (6e)$$

Eq. (6d) are the complementarity relations, which in this case express that either sliding or sticking conditions are active.